INTEGRALS & ITS APPLICATIONS

1 MARK QUESTIONS

1. Assertion (A): The are of the region bounded by the line y – 1 = x the x-axis and the ordinates x = –1 and x = 1 is 2 quare units.
Reason (R) : The are of the region bounded by the curve y = f(x), the x – axis and the ordinates x = a and x = b is given by $\int_a^b f(x)dx$.

[March, 2025]

2. $\int (x - 1)e^{-x}dx$ is equal to
(A) $(x - 2)e^{-x} + C$ (B) $xe^{-x} + C$
(C) $-xe^{-x} + C$ (D) $(x + 1)e^{-x} + C$

[March, 2023]

3. $\int 2^{2x}.\ 3^x dx$ is equal to:
(A) $\frac{12^x}{\log 12} + C$ (B) $\frac{2^{2x}.\ 3^x}{\log 2 \log 3} + C$
(C) $\frac{4.\ 6^x}{\log 6} + C$ (D) $12^x . \log 12 + c$

[July, 2024]

4. $\int \frac{1}{x + x\log x} dx$ is equal to:
(A) 1 + log x + C
(B) x + log x + C
(C) x log(1 + log x) + C
(D) log (1 + log x) + C

[July, 2023]

5. If the supply function is p = 4 + x, then the producer's surplus when 12 units are sold, is:
(A) 72 (B) 64 (C) 76 (D) 46

[July, 2023]

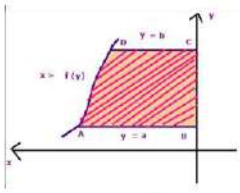
6. If $\int \frac{x + 2}{2x^2 + 6x + 5} dx = P \int \frac{4x + 6}{2x^2 + 6x + 5} dx + \frac{1}{2}\int \frac{dx}{2x^2 + 6x + 5}$ then the value of P is
(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{1}{6}$

[SQP 25-26]

7. $\int \frac{\log x}{x} dx$ equals
(A) $\frac{\log x}{2} + C$ (B) $\frac{(\log x)^2}{2} + C$
(C) log x + C (D) log (log x) + C

[SQP 24-25]

8. In the given figure, the area bounded by the curve $x = f(y)$, y -axis and abscissa $y = a$ and $y = b$ is equal to –



(A) $\int_a^b f(y)dy$ (B) $\int_a^b f(x)dx$
(C) $\int_a^b |f(y)|dy$ (D) $\int_a^b |f(x)|dx$

[SQP 22-23]

2 MARKS QUESTIONS

9. Evaluate: $\int_0^1 \frac{xe^x}{(x + 1)^2} dx$

[March, 2023]

10. If the marginal revenue function for output x is given by MR = $\frac{6}{(x + 2)^2} + 5$, find the total revenue function.

[July, 2022]

11. Evaluate: $\int_0^4 |x - 2| dx$

[July, 2022]

12. The marginal revenue function for a commodity is given by $MR = 9 + 2x - 6x^2$. Find the demand function.

[SQP 21-22]

13. The marginal cost of producing x pairs of tennis shoes is given by MC = $50 + \frac{300}{x + 1}$. If the fixed cost is ₹2000, find the total cost function.

[SQP 21-22]

3 MARKS QUESTIONS

14. Evaluate: $\int_0^2 x^2 dx$ ad hence show the region on the graph whose area it represents.

[March, 2024]

15. Evaluate: $\int_0^1 \frac{e^{-x}}{1 + e^x} dx$.

[March, 2024]

16. The supply function of a commodity is 100p = $(x + 20)^2$. Find the producer's surplus, when the market price is Rs 25.

[March, 2023]

17. Find : $\int \frac{2x^2 + 1}{x^2 - 3x + 2} dx$

[March, 2023]

18. Find : $\int \frac{dx}{(x + 1)^2(x^2 + 1)}$

[July, 2025]

19. The demand and supply function for a commodity are $P_d = 56 - x^2$ and $P_s = 8 + \frac{x^2}{3}$. Find the consumer's surplus at equilibrium price.

[July, 2022]

20. Evaluate: $\int_0^1 \log(1 + 2x)\, dx$

July, 2022]

21. Find: $\int \frac{x^3}{(x + 2)} dx$

[SQP 23-24]

22. Find: $\int (x^2 + 1) \log x\, dx$

[SQP 23-24]

23. The demand and supply functions under the pure market competition are $p_d = 16 - x^2$ and $p_s = 2x^2 + 4$ respectively, where p is the price and x is the quantity of the commodity. Using integrals find consumer's surplus.

[SQP 23-24]

24. The demand and supply functions under the pure market competition are $p_d = 56 - x^2$ and $p_s = 8 + \frac{x^2}{3}$ respectively, where p is the price and x is the quantity of the commodity. Using integrals find producer's surplus.

[SQP 23-24]

25. Evaluate: $\int \frac{dx}{(1 + e^x)(1 + e^{-x})}$

[SQP 22-23]

26. Evaluate: $\int x \log(1 + x^2)\, dx$

[SQP 22-23]

27. Under the pure market competition scenario, the demand function $p_d = \frac{8}{x + 1} - 2$ and supply function $p_s = \frac{x + 3}{2}$ respectively, where p is the price and x is the quantity of the commodity. Using integrals, find the producer's surplus.

[SQP 22-23]

28. The demand function p for maximising a profit monopolist is given by $p = 274 - x^2$ while the marginal cost is $4 + 3x$, for x units of the commodity. Using integrals, find the consumer surplus.

[SQP 22-23]

29. The supply function for a commodity is given by $p = x^2 + 4x + 3$, where x is the quantity supplied at the price p. Find the producers surplus when the price of the commodity is ₹48.

[SQP 21-22]

5 MARKS QUESTIONS

30. If the supply function is $p = 4 - 5x + x^2$, then find the produce's surplus when price is 18.

[March, 2025]

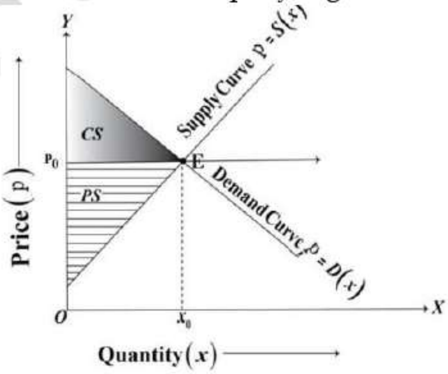
31. Find : $\int \frac{x^3}{x^4 + 3x^2 + 2} dx$

[July, 2024]

32. A company has approximated the marginal cost and marginal revenue functions for one of its products by $MC = 81 - 16x + x^2$ and $MR = 20x - 2x^2$ respectively. Determine the profit maximizing output and the total profit at the optimum output, assuming fixed cost as zero.

[SQP 25-26]

33. Supply and demand curves of a tyre manufacturer company is given below:

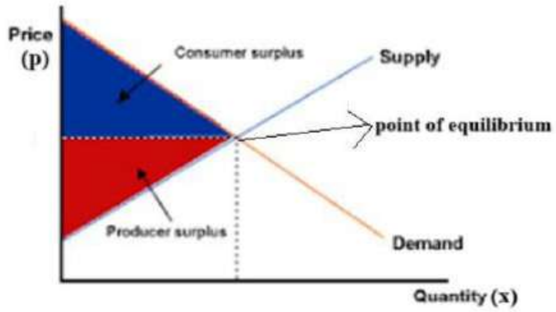


The above graph showing the demand and supply curves of a tyre manufacturer company which are linear. 'ABC' tyre manufacturer sold 25 units every month when the price of a tyre was ₹ 20000 per units and 'ABC' tyre manufacturer sold 125 units every month when the price dropped to ₹ 15000 per unit. When the price was ₹ 25000 per unit, 180 tyres were available per month for sale and when the price was only ₹ 15000 per unit, 80 tyres remained. Find the demand function. Also find the consumer surplus if the supply function is given to be $\boldsymbol{S(x) = 100x + 700}$

[SQP 24-25]

CASE-BASED QUESTIONS

34. In the grain market for wheat, the relationship between price and quantity demanded can be modelled using a linear demand function.



Suppose the following information is available from market data:

• At a price of ₹ 20 per kilogram, the quantity demanded is 400 tons.

• At a price of ₹ 25 per kilogram, the quantity demanded decreases to 200 tons.

Based on the above information, answer the following questions:

(i) Formulate the linear demand function based on the given data.

(ii) Suppose the supply function is given by $ps = -15 + x\ 20$, determine the equilibrium price and quantity.

(iii) (A) Using integration, calculate the consumer surplus at the equilibrium price.

OR

(B) Using integration, calculate the producer surplus at the equilibrium price.

[SQP 25-26]