PROBABILITY

1 MARK QUESTIONS

1. A pair of dice is thrown two times. If X represent the number of doubles obtained, then the expectation of X is

(A) $\frac{1}{6}$ (B) 1 (C) $\frac{1}{3}$ (D) $\frac{11}{36}$

[March, 2025]

2. If a random variable X has the probability distribution

$$P(X=x)=\begin{cases} k, if\ x=0 \\ 2k, if\ x=1\ or\ 2 \\ 0, otherwise, \end{cases}$$

Then the value of k is

(A) $\frac{1}{3}$ (B) $\frac{1}{5}$ (C) $\frac{1}{6}$ (D) $\frac{1}{4}$

[March, 2025]

3. A fair coin is tossed twice and outcomes are noted. If the random variable X represents the number of heads that appeared in the experiment, then the mathematical expectation of X is :

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $1\frac{1}{2}$

[March, 2024]

4. If for a Poisson variate X, P(X = k) = P(X = k + 1), then the variance of X is :

(A) k – 1 (B) k (C) k + 1 (D) k + 2

[March, 2024]

5. In a binomial distribution, n = 200 and p = 0.04. Taking Poisson distribution as an approximation to the binomial distribution :

Assertion (A) : Mean of Poisson distribution = 8.

Reason (R) : P (X = 4) = $\frac{512}{3e^8}$.

[March, 2024]

6. If X is a Poisson variable such that P(X = 1) = 2P(X = 2), then P(X = 0) is :

(A) e (B) $\frac{1}{e}$ (C) 1 (D) e^2

[March, 2023]

7. For a random variable X, E(X) = 3 and $E(X^2)$ = 11. The variance of X is :

(A) 8 (B) 5 (C) 2 (D) 1

[July, 2025]

8. If the mean and standard deviation of a binomial distribution are 12 and 2 respectively, then the value of the parameter p is :

(A) $\frac{5}{6}$ (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$

[July, 2025]

9. If the variance of a Poisson distribution is 2, then P(X = 2) is :

(A) $4e^2$ (B) $2e^2$ (C) $\frac{2}{e^2}$ (D) $\frac{4}{e^2}$

[July, 2025]

10. Normal distribution is symmetric about :

(A) Variance (B) Co-variance

(C) Mean (D) Standard deviation

[July, 2025]

11. A random variable X takes the values – 1, 0, 1. If its mean is 0.6 and P(X = 0) = 0.2, then P(X = 1) is :

(A) 0.7 (B) 0.5 (C) 0.4 (D) 0.3

[July, 2024]

12. One hundred identical coins each with probability p showing up heads are tossed once. If 0 < p < 1 and the probability of heads on 50 coins is equal to that of heads showing on 51 coins, then the value of p is :

(A) $\frac{1}{2}$ (B) $\frac{49}{101}$ (C) $\frac{50}{101}$ (D) $\frac{51}{101}$

[July, 2024]

13. The probability that a bomb dropped from a plane strikes the target is $\frac{4}{5}$. What is the probability that out of 6 bombs dropped, exactly 2 bombs strike the target ?

(A) $2\left(\frac{4}{5}\right)^5$ (B) $1-2\left(\frac{4}{5}\right)^5$ (C) $\frac{48}{5^5}$ (D) $\frac{64}{5^6}$

[July, 2024]

14. If 'm' is the mean of Poisson distribution, then P(r = 0) is given by

(A) e^{-m} (B) e^m (C) e (D) m^{-e}

[July, 2023]

15. Normal distribution is symmetric about :

(A) variance (B) standard deviation

(C) mean (D) covariance

[July, 2023]

16. A fair coin is tossed 6 times. Let X be the number of heads obtained. If $P(X = k) = P(X = k + 2)$, then the value of k is
(A) 4 (B) 3 (C) 2 (D) 1

[SQP 25-26]

17. Assertion (A): If a random variable X follows a binomial distribution with parameters n and p, then the mean of X is always less than its variance.
Reason (R): Probability of an event lies between 0 and 1 (0 and 1 included).

[SQP 25-26]

18. A player rolls one fair die. If the die shows an odd number, the player wins the value that appears on the die, else loses half the value that appears on it. The expected gain of the player is

(A) -0.5 (B) 0 (C) 0.5 (D) 1

[SQP 24-25]

19. In a large consignment of electric bulbs 5% of a batch of batteries are defective. A random sample of 80 is taken for inspection with replacement. Then the Variance of the number of defectives in the sample, is
(A) 3.6 (B) 3.8 (C) 4.555 (D) 8

[SQP 24-25]

20. If X is a Poisson variate such that 3P(X=2) = 2P(X=1) then the mean of the distribution is equal to
(A) $\frac{4}{3}$ (B) $\frac{3}{4}$ (C) $\frac{-4}{3}$ (D) $\frac{-3}{4}$

[SQP 23-24]

21. The data point of a normal variate with mean 12, standard deviation 4 and Z – score 5 is
(A) 28 (B) 304 (C) 34 (D) 32

[SQP 23-24]

22. Assertion (A): The probability of getting 6 heads when a unbiased coin is tossed 10 times is $C(10,6)\left(\frac{1}{2}\right)^{10}$
Reason (R) In a Binomial distribution the probability is given by P(X= r) = C (n, r)$(p)^r(q)^{n-r}$

[SQP 23-24]

23. Assertion (A): Kuhu and Beena are two equally capable badminton players.
Probability that Beena will beat Kuhu in 3 games out of 4 is 25%
Reason (R): The probability of r successes in n trials, denoted by P(X = r) is given by P(X = r) = nCr $p^r q^{n-r}$, r = 0, 1, . . . , n where p denotes success and q denotes failure in each trial.

[SQP 22-23]

24. Let X denotes the number of hours a student devotes to self-study during a randomly selected school day. The probability that X takes the value x, where k is some unknown constant is

$$P(X = x) = \begin{cases} k & if\ x = 0 \\ kx & if\ x = 1\ or\ 2 \\ k(5-x) & if\ x = 3\ or\ 4 \\ 0 & otherwise \end{cases}$$

The probability that a student studies at least 3 hours on a particular day is

(A) $\frac{1}{7}$ (B) $\frac{2}{7}$ (C) $\frac{3}{7}$ (D) $\frac{1}{2}$

[SQP 21-22]

25. For a Poisson distribution with mean λ, $\sum_{k=0}^{\infty} \frac{e^{-\lambda}\lambda^k}{k!}$ is equal to
(A) –1 (B) 0 (C) 0.5 (D) 1

[SQP 21-22]

26. A TV manufacturer tests a random sample of 6 picture tubes to determine any defect. Past experience suggests the probability of defective picture tube is 0.05. The probability that there is at least one defective picture tube in the sample is

(A) $\left(\frac{19}{20}\right)^6$ (B) $1 - \left(\frac{19}{20}\right)^6$

(C) $1 - \left[\left(\frac{19}{20}\right)^6 + \frac{3}{10}\left(\frac{19}{20}\right)^5\right]$ (D) $\left(\frac{1}{20}\right)^6$

[SQP 21-22]

27. A sales promotion company sells tickets for ₹100 each to win a prize of ₹5 lakhs. If a person buys one of the 10,000 tickets sold, then his expected gain in rupees is

(A) -50 (B) 0 (C) 50 (D) 100

[SQP 21-22]

28. During a pandemic, 10% of the patients who have the disease get complications. If 100 patients of a locality get infected by the disease, then the standard deviation of the number of patient getting complications is:
(A) 10 (B) 9 (C) 6 (D) 3
[SQP 21-22]

29. An insurance company has found that 50% of its claims are for damages resulting from accidents. The probability that a random sample of 10 claims will contain fewer than 2 for accidents is
(A) $\frac{1}{1024}$ (B) $\frac{5}{512}$ (C) $\frac{11}{1024}$ (D) $\frac{15}{1024}$
[SQP 21-22]

30. An electrical supplier distributor has found the daily demand for fluorescent light bulbs is normally distributed with a mean of 432 and standard deviation of 86. Find the probability that the demand on a particular day exceeds 518 bulbs.
(A) 0.1587 (B) 0.3413 (C) 0.7587 (D) 0.8413
[SQP 21-22]

31. The value of mortgage loans made by a certain bank is normally distributed with mean of ₹36 lacs and a standard deviation of ₹12 lacs. The probability that a randomly selected mortgage loan is less than 54 lacs is
(A) 85.26% (B) 93.32% (C) 97.42% (D) 98.04%
[SQP 21-22]

32. A river passing near a town floods it on an average twice every 10 years. Assuming Poisson distribution find the probability that the town faces flooding at least once in 10 years.
(A) 0.0198 (B) 0.1353 (C) 0.5657 (D) 0.8647
[SQP 21-22]

33. The height of certain species of plant is normally distributed with mean of 20 cm and standard deviation of 4 cm. what is the probability that the height of a plant chosen at random lies between 10 cm and 30 cm
(A) 0.0062 (B) 0.5341 (C) 0.9876 (D) 0.9938
[SQP 21-22]

2 MARKS QUESTIONS

34. If a fair coin is tossed 6 times, find the probability of getting at least 4 heads.
[March, 2025]

35. Given that mean of a normal variate X is 9 and standard deviation is 3, then find:
(a) the z-score of the data point is 15
(b) the data point if its z-score is 4.
[March, 2025]

36. Suppose 2% of the items made by a factory are defective. Find the probability that there are 3 defective items in a sample of 100 items selected at random. (Given $e^{-2} = 0.135$)
[March, 23]

37. Find the probability distribution of a number of successes in two tosses of a die, where a success is defined as getting a number greater than 4.
[July, 2023]

38. If X is a normal variate with mean (m) = 70 and standard deviation (s) = 5, then find P(X > 75). (Given : P(0 < Z < 1) = 0.3413)
[July, 2024]

39. If X is a Poisson variate such that P(X = 0) = P(X = 1) = α, then show that $\alpha = e^{-1}$.
[July, 2024]

40. It is given that 2% of screws manufactured by a company are defective. Using Poisson distribution, find the probability that a packet of 100 screws contains no defective screw. (Given : $e^{-2} = 0\ 14$)
[July, 2023]

41. If the standard deviation of a Poisson variable X is 3 , then find P (X > 0). [Use : $e^{-3} = 0.05$]
[July, 2023]

42. A random variable *X* has the following probability distribution:

$$P(X = x) = \begin{cases} a\,x, & \text{if } x = 1, 2, 3 \\ b, & \text{if } x = 4 \end{cases}$$

where a and b are constants. If it is given that the mean of the distribution is 2.8, then find the values of a and b.

[SQP 25-26]

43. A small town experiences an average of 2 power outages per month. Assuming the number of power outages follows a Poisson distribution, find the probability that in a given month, there will be exactly 3 power outages, given that there will be at least one power outage (Use $e^{-2} = 0.14$).

[SQP 25-26]

44. A certain type of electronic component fails at a rate of 2 failures per 1000 hours of operation. Assuming the failures follow a Poisson distribution, find the probability that in a 5000-hour operation, there will be more than 1 failure. (Use $e^{-10} = 4.54 \times 10^{-5}$)

[SQP 25-26]

45. The incidence of occupational disease in an industry is such that the workers have a 20% chance of suffering from it. What is the probability that out of six workers 4 or more will catch the disease?

[SQP 24-25]

46. The lifetime of an item produced by a machine has a normal distribution with mean 12 months and standard deviation of 2 months. Find the probability of an item produced by this machine will last (i) less than 7 months (ii) between 7 and 14 months. (Given P(Z < 2.5) = 0.9938, P(Z < 1) = 0.8413)

[SQP 24-25]

3 MARKS QUESTIONS

47. Two numbers are selected at random (without replacement) from first six positive integers. Let X denotes the smaller of the two numbers obtained. Calculate the mathematical expectation of X.

[March, 2025]

48. If the mean and variance of a binomial distribution are $\frac{4}{3}$ and $\frac{8}{9}$ respectively, then find P(x = 1).

[March, 2025]

49. The mortality rate for a certain disease if 0.007. Using Poisson distribution, calculate the probability for 2 deaths in a group of 400 people. [Use $e^{-2.8} = 0.0608$]

[March, 2025]

50. Given that the scores of a set of candidates on an IQ test are normally distributed. If the IQ test has a mean of 100 and a standard deviation of 10, determine the probability that a candidate who takes the test will score between 90 and 110. [Given P (Z < 1) = 0 ·8413 and P (Z < – 1) = 0 ·1587]

[March, 2024]

51. An unbiased die is thrown again and again until three sixes are obtained. Find the probability of obtaining a third six in the sixth throw of the die.

[March, 2024]

52. A fair coin is tossed 9 times. Find the probability of getting.

(i) exactly 5 tails;
(ii) at least 5 tails;
(iii) at most 5 tails.

[July, 2023]

53. Let X denote the number of hours a person watches T.V. during a randomly selected day. The probability that X can take the values xi, has the following form, where k is some unknown constant

$$P(X = x_i) = \begin{cases} 0{\cdot}2, & \text{if } x_i = 0 \\ kx_i, & \text{if } x_i = 1 \text{ or } 2 \\ k(5 - x_i), & \text{if } x_i = 3 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the value of k.
(ii) Find : P(X = 2), P(X 2) and P(X 2).

[July, 2023]

54. 2000 students appeared in an examination. Distribution of marks is assumed to be normal

with mean 30 and standard deviation 6 25. How many students are expected to get marks

(i) between 20 and 40 ?

(ii) less than 25 ?

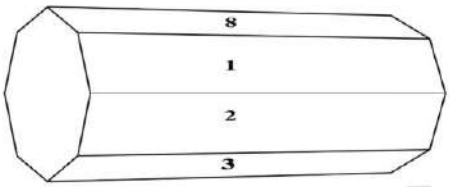
[Use : $P(0 \leq z \leq 1.6) = 0.4452$, $P(0 \leq z \leq 0.8) = 0.2881$]

[July, 2023]

55. A traffic engineer records the number of bicycle riders that use a particular cycle track. He records that an average of 3.2 bicycle riders use the cycle track every hour. Given that the number of bicycles that use the cycle track follow a Poisson distribution, what is the probability that 2 or less bicycle riders will use the cycle track within an hour? Also find the mean expectation and variance for the random variable. (Given $e^{-3.2} = 0.041$)

[SQP 24-25]

56. An octagonal prism is a three-dimensional polyhedron bounded by two octagonal bases and eight rectangular side faces. It has 24 edges and 16 vertices.



The prism is rolled along the rectangular faces and number on the bottom face (touching the ground) is noted. Let X denotes the number obtained on the bottom face and the following table gives the probability distribution of X .

X:	1	2	3	4	5	6	7	8
$P(X)$:	p	$2p$	$2p$	p	$2p$	p^2	$2p^2$	$7p^2+p$

On the above context, answer the following questions. (i) Find the value of p. (ii) Find the mean, E(X).

[SQP 24-25]

57. If the probability of success in a single trial is 0.01 , how many minimum number of Bernoulli trials must be performed in order that the probability of at least one success is 0.5 or more?

(Use $\log_{10} 2 = 0.3010$ and $\log_{10} 99 = 1.9956$)

[SQP 24-25]

5 MARKS QUESTIONS

58. Let X denote the number of hours a Class 12 student studies during a randomly selected school day. The probability that X can take the values x, for an unknown constant 'k' :

$$P(X=x) = \begin{cases} 0.1 \ , if\ x = 0 \\ kx \ , if\ x = 1\ or\ 2 \\ k(5-x) \ , \ if\ x = 3\ or\ 4 \end{cases}$$

(i) Find the value of k.

(ii) Determine the probability that the student studied for at least 2 hours.

(iii) Determine the probability that the student studied for at most 2 hours.

[March, 2024]

59. A river near a small-town floods and overflows twice in every 10 years on an average. Assuming that the Poisson distribution is appropriate, what is the mean expectation? Also, calculate the probability of 3 or less overflows and floods in a 10-year interval. [Given $e^{-2} = 0\cdot13534$]

[March, 2024]

60. An unbiased die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.

[July, 2024]

61. An aptitude test for selecting officers in a bank is conducted on 1000 candidates. The mean score obtained is 42 and the standard deviation of score is 24. Assuming normal distribution for the scores, find :

(i) the number of candidates whose scores exceed 60;

(ii) the number of candidates whose scores lie between 30 and 60.

[Given : $P(0 < Z < 0.75) = 0.2734$; $P(0 < Z < 0.5) = 0.1915$]

[July, 2024]

62. On doing the proof reading of a book on an average 4 errors in 10 pages were detected. Using Poisson's distribution find the probability of (i) No error and (ii) one error in 1000 pages of first printed edition of the book (Given $e^{-0.4} = 0.6703$)

[SQP 23-24]

63. How many times Sunil toss a fair coin so that the probability of getting at least one head is more than 90 %?

[SQP 23-24]

64. It is known that 3% of plastic buckets manufactured in a factory are defective. Using the Poisson distribution on a sample of 100 buckets, find the probability of: (i) Zero defective buckets (ii) At most one bucket is defective [Use $e^{-3} = 0.049$]

[SQP 22-23]

65. In a math aptitude test, student scores are found to be normally distributed having mean as 45 and standard deviation 5. What percentage of students have scores - (i) more than the mean score? (ii) between 30 and 50?

[SPQ 22-23]

CASE-BASED QUESTIONS

66. A factory produces bulbs, of which 6% are defective bulbs in a large bulk of bulbs.

Based on the above information, answer the following questions :

(i) Find the probability that in a sample of 100 bulbs selected at random, none of the bulbs is defective.

(Use : $e^{-6} = 0\cdot0024$)

(ii) Find the probability that the sample of 100 bulbs has exactly two defective bulbs.

(iii) (a) Find the probability that the sample of 100 bulbs will include not more than one defective bulb.

OR

(iii) (b) Find the mean and the variance of the distribution of number of defective bulbs in a sample of 100 bulbs.

[March, 2023]

67. According to an educational board survey, it was observed that class XII students apply at least one to four weeks ahead of college application deadlines. Let X represent the week when an average student applies ahead of a college's application deadline and the probability of the student to get admission in the college P(X = x) is given as follows :

$$P(X=x)=\begin{cases}\frac{kx}{6}, & \text{when } x = 0, 1 \text{ or } 2\\ \frac{(1-k)x}{6}, & \text{when } x = 3\\ \frac{kx}{2}, & \text{when } x = 4\\ 0, & \text{when } x > 4\end{cases}$$

where k is a real number.

Based on the above information, answer the following questions :

(I) Determine the value of k.

(II) What is the probability that Mahesh will get admission in the college, given that he applied at least 3 weeks ahead of application deadline ?

(III) (a) Calculate the mathematical expectation of number of weeks taken by a student to apply ahead of a college's application deadline.

OR

(b) To promote early admissions, the college is offering scholarships to the students for applying ahead of deadline as follows : ₹ 50,000 for applying 4 weeks ahead, ₹ 20,000 for applying 3 weeks ahead, ₹ 12,000 for applying 2 weeks ahead, and ₹ 9,600 for applying 1 week ahead. Determine the expected scholarship offered by the college.

[July, 2025]

68. The mathematics scores of a group of 500 students follow a normal distribution with a mean of 75 and a standard deviation of 8.

Based on this data, answer the following questions:

(i) What percentage of students scored below 75 marks?

(ii) Find the number of students who scored more than 82 marks.

(iii) (A) Calculate the number of students scoring between 67 and 83 marks.

OR

(iii) (B) The top 10% of students are awarded a scholarship. The Z-score for the 90th percentile is 1.28. Determine the minimum score required to qualify for the scholarship.

Use $P(Z < 0.875) = 0.8092$, $P(Z < 1) = 0.8413$, $P(Z < -1) = 0.1587$

[SQP 25-26]

69. Let X denote the number of hours a person watches television during a randomly selected day. The probability that X can take the values x_i, has the following form, where 'k' is some unknown constant.

$$P(X = x_i) = \begin{cases} 0.2, & \text{if } x_i = 0 \\ kx_i, & \text{if } x_i = 1 \text{ or } 2 \\ k(5 - x_i), & \text{if } x_i = 3 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the value of k.

(ii) What is the probability that a person watches two hours of television on a selected day?

(iii) What is the probability that the person watches at least two hours of television on a selected day ?

OR

(iv) What is the probability that the person watches at most two hours of television on a selected day ?

[SQP 23-24]

70. According to an educational board survey, it was observed that class XII students apply at least one to four weeks ahead of colleges application deadline. Let X represent the week when an average student applies ahead of a college's application deadline and the probability of student to get admission in the college $P(X = x)$ is given as follows:

$$P(X = x) = \begin{cases} \frac{kx}{6} & \text{when } x = 0, 1 \text{ or } 2 \\ \frac{(1-k)x}{6} & \text{when } x = 3 \\ \frac{kx}{2} & \text{when } x = 4 \\ 0 & \text{when } x > 4 \end{cases}$$

Where k is a real number. Based on the above information, answer the following questions. Show steps to support your answers.

(i) Find the value of k.

(ii) What is the probability that Sonali will get admission in the college, given that she applied at least 2 weeks ahead of application deadline?

(iii) Calculate the mathematical expectation of number of weeks taken by a student to apply ahead of a college's application deadline.

OR

(iii)To promote early admissions, the college is offering scholarships to the students for applying ahead of deadline as follows: ₹ 50000 for applying 4 weeks early, ₹ 20000 for applying 3 weeks early, ₹ 12000 for applying 2 weeks early, and ₹ 9600 for applying 1 week early What is the expected scholarship offered by the college?

[SQP 22-23]