DIFFERENTIATION & APPLICATIONS OF DERIVATIVES

1 MARK QUESTIONS

1. The rate of change of population P(t) with respect to time (T) , where α, β are the constant birth and death rates, respectively, is

(A) $\frac{dP}{dt} = (\alpha + \beta)P$ (B) $\frac{dP}{dt} = (\alpha - \beta)P$

(C) $\frac{dP}{dt} = \frac{\alpha + \beta}{P}$ (D) $\frac{dP}{dt} = \frac{\alpha - \beta}{P}$

[March, 2025]

2. If the cost function and revenue function of x times are respectively given as $C(x) = 100 + 0.015x^2, R(x) = 3x$, then the value of x for maximum

(A) 30 (B) 100 (C) 150 (D) 200

[March, 2025]

3. The demand function of a monopolist is given by p = 30 + 5x – 3x², where x is the number of units demanded and p is the price per unit. The marginal revenue when 2 units are sold, is

(A) Rs 28 (B) Rs 23 (C) Rs 1 (D) Rs 14

[March, 2025]

4. If $y = e^{-2x}$, then $\frac{d^3y}{dx^3}$ is equal to

(A) $2e^{-2x}$ (B) e^{-4x} (C) $4e^{-4x}$ (D) $-8e^{-2x}$

[March, 2024]

5. The function f(x) = $x^2 - x + 1$ is

(A) increasing in (0, 1)

(B) decreasing in (0, 1)

(C) increasing in (0, 0.5) and decreasing in (0.5. 1)

(D) increasing in (0.5, 1) and decreasing in (0,0.5)

[March, 2024]

6. Assertion (A) : The function $f(x) = x^2 - x + 1$ is strictly increasing on (–1, 1).

Reason (R) : If f(x) is continuous on [a, b] derivable on (a, b), then f(x) is strictly increasing on [a, b] if f′(x) > 0 for all x ϵ (a, b).

[March, 2024]

7. The relation between 'marginal cost' and 'average cost' of producing 'x' units of a product is:

(A) $\frac{d(AC)}{dx} = x(MC - AC)$

(B) $\frac{d(AC)}{dx} = x(AC - MC)$

(C) $\frac{d(AC)}{dx} = \frac{1}{x}(AC - MC)$

(D) $\frac{d(AC)}{dx} = \frac{1}{x}(MC - AC)$

[March, 2023]

8. Assertion (A): The function $f(x) = (x + 2)e^{-x}$ is increasing in the interval (–1, ∞).

Reason (R) : A function f(x) is increasing, if f′(x) > 0.

[March, 2023]

9. If $y = x^y$, then $\frac{dy}{dx}$ is :

(A) x^y (log x + 1) (B) $\frac{y^2}{x(1 + y\log x)}$

(C) x^y (log x - 1) (D) $\frac{y^2}{x(1 - y\log x)}$

[July,2025]

10. The function f(x) = a^x is increasing on R, if :

(A) a > 0 (B) a > 1

(C) a < 0 (D) 0 < a < 1

[July, 2025]

11. A function f : R → R is defined as f(x) = x³ + 1. The function f has :

(A) no maximum value

(B) no minimum value

(C) both maximum and minimum values

(D) neither minimum nor maximum

[July, 2025]

12. The relation between 'marginal cost' and 'average cost' of producing 'x' units of a product is:

(A) $\frac{d(AC)}{dx} = x(MC - AC)$

(B) $\frac{d(AC)}{dx} = x(AC - MC)$

(C) $\frac{d(AC)}{dx} = \frac{1}{x}(AC - MC)$

(D) $\frac{d(AC)}{dx} = \frac{1}{x}(MC - AC)$

[July, 2025]

13. If x + y = 8, then the maximum value of (xy) is :

(A) 12 (B) 16 (C) 20 (D) 24

[July, 2024]

14. The demand curve for a monopolist is given by x = 100 – 4p. The value of x for which MR = 0, is :

(A) 25 (B) 30 (C) 45 (D) 50

[July, 2024]

15. If x + y = 8, then the maximum value of (xy) is :

(A) 12 (B) 16 (C) 20 (D) 24

[July, 2023]

16. The function $f(x) = x^x$, $x > 0$ is decreasing in the interval :

(A) $(-\infty, e)$ (B) $(0, e)$

(C) $\left(0, \frac{1}{e}\right)$ (D) $\left[\frac{1}{e}, \infty\right)$

[July, 2023]

17. If $x = at^2$, and $y = 2at$, then at $t = 2$ the value of $\frac{d^2y}{dx^2}$ is

(A) $\frac{-1}{16a}$ (B) $\frac{-1}{16}$

(C) $\frac{1}{8a}$ (D) $\frac{-1}{4}$

[SQP 25-26]

18. Let the cost function for a manufacturer is given by $C(x) = \frac{x^3}{3} - x^2 + 2x$ (in rupees). Which of the following statement is correct based on the above information?

(A) The marginal cost decreases from 0 to 1 and then increases onwards.

(B) The marginal cost increases from 0 to 1 and then decreases onwards.

(C) Marginal cost decreases as production level increases from zero.

(D) Marginal cost increases as production level increases from zero.

[SQP, 24-25]

19. The absolute minimum value of the function $f(x) = 4x - \frac{1}{2}x^2$ in the interval $[-2, 4.5]$ is:

(A) –8 (B) –9

(C) –10 (D) –16

[SQP 24-25]

20. Assertion (A) : The maximum profit that a company makes if profit function is given by $P(x) = 41 + 24x - 8x^2$; where 'x' is the number of units and P is the profit is Rs. 59.

Reason (R): The profit is maximum at $x = a$ if $P'(a) = 0$ and $P''(a) > 0$.

[SQP 23-24]

21. The total cost function is given by $C(x) = x^2 + 30x + 1500$. The marginal cost when 10 units are produced is

(A) Rs 20 (B) Rs 30

(C) Rs 50 (D) Rs 70

[SQP 21-22]

22. The function $y = \frac{1}{x}$ is strictly decreasing in the interval (s)

(A) $(0, \infty)$ only (B) $(-\infty, 0)$ only

(C) $(-\infty, 0)$ and $(0, \infty)$ (D) R

[SQP 21-22]

23. If $x = at^2$, and $y = 2at$, then value of $\frac{d^2y}{dx^2}$ is

(A) $\frac{-1}{2at^3}$ (B) $\frac{-1}{2at^2}$

(C) $\frac{1}{t^2}$ (D) $\frac{-2a}{t}$

[SQP 21-22]

24. The variable cost of producing x units is $V(x) = x^2 + 2x$. If the company incurs a fixed cost of Rs 10,000, then the level of output where the average cost is minimum is

(A) 10 units (B) 50 units

(C) 100 units (D) 200 units

[SQP 21-22]

25. The demand function of a toy is, $x = 75 - 3p$ and its total cost function is $TC = 100 + 3x$. For maximum profit the value of x is

(A) 33 (B)31

(C) 29 (D) 24

[SQP 21-22]

2 MARKS QUESTIONS

26. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at the which the ordinate is changing 8 times as fast as abscissa.

[March, 2023]

27. If $y = \left(x + \sqrt{x^2+1}\right)^p$, then prove that $(x^2 + 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - p^2y = 0$.

[July, 2024]

3 MARKS QUESTIONS

28. Find all the points of local maxima and local minima for the function $f(x) = x^3 - 6x^2 + 9x - 8$.

[March, 2023]

29. Prove that the function $f(x) = x^2 - x + 1$ is neither strictly increasing nor strictly decreasing on the interval $(-1, 1)$.

[July, 2025]

30. Find $\frac{dy}{dx}$, if $y^x + x^y + x^x = a^b$.

[July, 2025]

31. A curve given by $y = px^3 + qx^2$, where p and q are constants, has one of its critical points at

(1, –1). Find the values of p and q. Also find the other critical point.

[SQP 25-26]

32. The total revenue function for a commodity is $R = 15x + \frac{x^2}{3} - \frac{x^4}{36}$. Show that the point at which average revenue is maximum, the average revenue and marginal revenue has the same value.

[SQP 25-26]

33. Find the intervals in which the function f(x) = $2x^3 - 9x^2 + 12x - 5$ is increasing or decreasing.

[SQP 23-24]

34. Find the interval(s) in which the function $f(x) = \frac{x^4}{4} - 2x^3 + \frac{11x^2}{2} - 6x$, is strictly increasing and strictly decreasing.

[SQP 22-23]

5 MARKS QUESTIONS

35. Find all the points of local maxima and local minima of the function $f(x) = \frac{-3x^4}{4} - 8x^3 - \frac{45}{2}x^2 + 105$.

[March, 2024]

36. Find the intervals in which the following function f(x) is strictly increasing or strictly decreasing: $f(x) = 20 - 9x + 6x^2 - x^3$.

[March, 2024]

37. Divide a number 15 into two parts such that the square of one part multiplied with the cube of the other part is maximum.

[March, 2023]

38. Find a point on the curve $y^2 = 2x$ which is nearest to the point (1, 4).

[March, 2023]

39. A given rectangular area is to be fenced off in a field whose length lines along a straight river. If no fencing is needed along the river, show that the least length of fencing will be required when the length of the rectangular area is twice its breadth.

[July, 2025]

40. A wire of length 36 m is to be cut into two pieces. One of the pieces is to be made a square and the other, a circle. What would be the lengths of the two pieces, so that the combined area of the square and the circle is minimum?

[July, 2024]

41. Determine for what value of x, the function f(x) = $x^4 - \frac{x^3}{3}$ is strictly increasing or strictly decreasing.

[July, 2024]

42. A firm has the following total cost and demand functions : C(x) = $\frac{x^3}{3} - 7x^2 + 111x + 50$ and x = 100 – p.

(i) Find the total revenue function in terms of x.

(ii) Find the total profit function P in terms of x.

(iii) Find the profit maximizing level of output of x.

(iv) What is the maximum profit, taking rupee as a unit of money ?

[July, 2023]

43. A company produces a certain commodity with Rs 2400 fixed cost. The variable cost is estimated to be 25% of the total revenue received on selling the product at a rate of Rs 8 per unit. Find the following:

(i) Cost function

(ii) Revenue Function

(iii) Breakeven point

(iv) Profit Function

[SQP 23-24]

44. The production manager of a company plans to include 180 sq cm of actual printed matter in each page of book under production. Each page should have a 2.5 cm wide margin along the top and bottom and 2 cm wide margin along the sides. What are the most economical dimensions of each printed page?

[SQP 23-24]

45. An event management company charges Rs 4800 per guest, for a bulk booking for 100 guests. In addition, it offers a discount of Rs 200 for each group of 10 guests over and above 100 guest booking. What is the number of guests that will maximise the amount of money the company receives on a booking? What is the maximum profit on such booking?

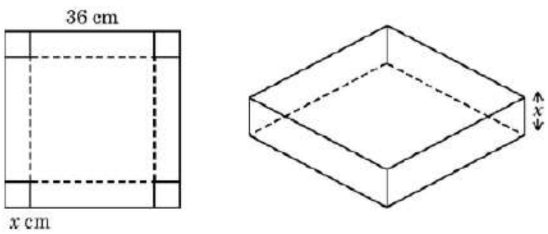
[SQP, 22-23]

46. To manufacture 'x' number of dolls, a company's total cost function C(x) is given by C(x) = 100 + 0.025x and the total revenue function R(x) is described as R(x) = 5x. Given that C(x) and R(x) are in thousand rupees, what number of dolls shall be manufactured to maximise profit of the company? What is the maximum profit?

[SQP, 22-23]

CASE-BASED QUESTIONS

47. A man has an expensive square-shaped piece of golden board of side 36 cm. He wants to turn it into a box without top by cutting a square from each corner and folding the flaps. Let x cm be the side of square, which is cut from each corner.



Based on the above information, answer the following questions:

(i) Find the expression for the volume (V) of open box in terms of x.

(ii) Find $\frac{dV}{dx}$.

(iii) Find the value of x for which the volume (V) is maximum.

OR

(iii) Find the maximum volume of open box. (2)

[March, 2025]

48. Read the following passage and answer the questions given below:

'In an elliptical sports field, the authority wants to design a rectangular soccer field with the maximum possible area. The sports field is given by the graph of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.'

(i) If the length and breadth of the rectangular soccer field be 2x and 2y respectively, then find the area function A(x) in terms of x.

(ii) Find the critical point(s) of the area function A(x).

(iii) Using first derivative test, find the length 2x and breadth 2y of the soccer field (in terms of a and b) that maximize the area

OR

Using second derivative test, find the length 2x and breadth 2y of the soccer field (in terms of a and b) that maximize the area. [2m]

[July, 2024]

49. A company notes that higher sales of a particular item, which it produced, is achieved by lowering the price charged. As a result, the total revenue from the sales at first rises as the number of units sold increases, reaches the highest point, and then falls off. The pattern of revenue is described by the relation $y = 40000 - (x - 2000)^2$, where y is the total revenue and x is the number of units sold.

Based on the above information, answer the following questions:

(i) Find what number of units solve maximizes total revenue. [2M]

(ii) What is the amount of this maximum revenue? [1M]

(iii) What would be the total revenue if 2500 units were sold? [1M]

[July, 2023]

50. A student Shivam is running on a playground along the curve given by $y = x^2 + 7$. Another student Manita standing at point (3, 7) on playground wants to hit Shivam by paper ball when Shivam is nearest to Manita.

Based on above information, answer the following questions:

(i) Let at any instance while running along the curve $y = x^2 + 7$, Shivam's position be (x, y). Find the expression of the distance (D) between Shivam and Manita in terms of 'x'.

(ii) Find the critical point(s) of the distance function.

(iii) What is the distance between Shivam and Manita when they are at least distance from each other.

OR

Find the position of Shivam when he is closest to Manita. [2M]

[SQP, 24-25]