4. DETERMINANTS

1 MARK QUESTIONS

1. (C) $\begin{bmatrix} \frac{1}{6} & \frac{1}{6} \\ \frac{-1}{3} & \frac{2}{3} \end{bmatrix}$
2. (D) ±3
3. (A) 0
4. (A) 0
5. (d) –81
6. (A) –1
7. (B) $(A^2)^{-1} = (A^{-1})^2$
8. (B) $k \neq \frac{4}{3}$
9. (C) A is true and R is false
10. (B) 0.5
11. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
12. (C) 11
13. (D) 6400
14. (B) Both Assertion (A) and Reason (R) are true and (R) is not the correct explanation of Assertion (A).
15. (A) 108
16. (D) 4
17. (A) 0
18. (C) -3

2 MARKS QUESTIONS

19. Solution

Sol.

Here, $\mathbf{D} = \begin{vmatrix} 2 & 3 \\ 11 & -5 \end{vmatrix} = -43 \neq 0$ $\left(\frac{1}{2}\right)$

$\mathbf{D_1} = \begin{vmatrix} 5 & 3 \\ 6 & -5 \end{vmatrix} = -43$ $\left(\frac{1}{2}\right)$

$\mathbf{D_2} = \begin{vmatrix} 2 & 5 \\ 11 & 6 \end{vmatrix} = -43$ $\left(\frac{1}{2}\right)$

So, $\mathbf{x_1} = \frac{D_1}{D} = \frac{-43}{-43} = 1,\ \mathbf{x_2} = \frac{D_2}{D} = \frac{-43}{-43} = 1$ $\left(\frac{1}{2}\right)$

20. Solution

Sol.

Here, $D = \begin{vmatrix} 2 & -1 \\ 3 & 5 \end{vmatrix} = 13$ $\left(\frac{1}{2}\right)$

$D_1 = \begin{vmatrix} 17 & -1 \\ 6 & 5 \end{vmatrix} = 91$ $\left(\frac{1}{2}\right)$

$D_2 = \begin{vmatrix} 2 & 17 \\ 3 & 6 \end{vmatrix} = -39$ $\left(\frac{1}{2}\right)$

Thus, $x = \frac{D_1}{D} = 7;\ y = \frac{D_2}{D} = -3$ $\left(\frac{1}{2}\right)$

21. Solution

Sol. A is singular gives

$$\begin{vmatrix} x+1 & -3 & 4 \\ -5 & x+2 & 2 \\ 4 & 1 & x-6 \end{vmatrix} = 0$$ $(\frac{1}{2})$

i.e. $(x + 1) [(x + 2) (x - 6) - 2] + 3[- 5x + 30 - 8] + 4[- 5 - 4x - 8] = 0$

i.e. $(x + 1) (x^2 - 4x - 14) - 15x + 66 - 52 - 16x = 0$

i.e. $x^3 - 3x^2 - 49x = 0$ **(1)**

$x = 0, \frac{3\pm\sqrt{205}}{2}$

Hence, x = 0 is the only integral value. $(\frac{1}{2})$

22. Solution

Expanding C_1, we get $\Delta = 1(2x^2 + 4) - 2(-4x - 20) = 86$	1
$\Rightarrow x^2 + 4x - 21 = 0$ $\therefore x = 3, -7$	1

3 MARKS QUESTIONS

23. Solution

Sol. $|A| = 1 \neq 0$ **1**

$\therefore$ A is invertible and so A^{-1} exists.

adj A $= \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}' = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$ **1½**

$\Rightarrow A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$ **½**

24. Solution

Ans

Here $|A| = 1(3-0) - 2(-1-0) - 2(2-0)$

$= 3 + 2 - 4 = 1$ 1

$A_{11} = 3$, $A_{12} = 1$, $A_{13} = 2$

$A_{21} = 2$, $A_{22} = 1$, $A_{23} = 2$

$A_{31} = 6$, $A_{32} = 2$, $A_{33} = 5$

$$\text{cofactor matrix} = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}$$ 1

$$\Rightarrow adjA = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} adjA = \frac{1}{1} \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$ 1

5 MARKS QUESTIONS

25. Solution

Sol.

Here, $|A| = -(-4-3) - (12+1) + 2(9-1)$

$= 7 - 13 + 16 = 10 \neq 0$ **(1)**

$$\Rightarrow \text{adj(A)} = \begin{bmatrix} -7 & -13 & 8 \\ 2 & -2 & 2 \\ 3 & 7 & -2 \end{bmatrix}^T = \begin{bmatrix} -7 & 2 & 3 \\ -13 & -2 & 7 \\ 8 & 2 & -2 \end{bmatrix}$$ **($2\frac{1}{2}$)**

$$\text{Hence } A^{-1} = \frac{1}{10} \begin{bmatrix} -7 & 2 & 3 \\ -13 & -2 & 7 \\ 8 & 2 & -2 \end{bmatrix}$$ **($\frac{1}{2}$)**

$$AA^{-1} = \frac{1}{10} \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix} \begin{bmatrix} -7 & 2 & 3 \\ -13 & -2 & 7 \\ 8 & 2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ **(1)**

26. Solution

Sol.

The matrix equation AX = B is

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$ **($\frac{1}{2}$)**

$\|A\| = 10$	**(1)**
$\text{adj } A = \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}' = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$	**(2)**
Here $A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$	**($\frac{1}{2}$)**
So, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$	**(1)**
Thus, x = 2, y = – 1, z = 1	

27. Solution

Sol.	$\|A\| = 2(-4+4) + 3(-6+4) + 5(3-2) = -1 \neq 0$	½
	$\therefore A^{-1}$ exists	
	Now, $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$	**1**
	or $\quad AX = B \Rightarrow X = A^{-1}B$	
	$\text{adj}(A) = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$	**1½**
	$A^{-1} = \frac{1}{\|A\|}\text{adj}(A) = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$	½
	$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$	**1**
	So, $x = 1, y = 2, z = 3$	½

28. Solution

Ans(i)	$(x-50)(y+50) = xy \Rightarrow x - y = 50$	½
	and $(x-10)(y-20) + 5300 = xy \Rightarrow 2x + y = 550$	½
Ans(ii)	$\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$	1

	where, $A=\begin{bmatrix}1 & -1\\ 2 & 1\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}50\\ 550\end{bmatrix}$	
Ans (iii) (a)	$\lvert A\rvert=3$ and $adjA=\begin{bmatrix}1 & 1\\ -2 & 1\end{bmatrix}$ $\therefore A^{-1}=\frac{1}{3}\begin{bmatrix}1 & 1\\ -2 & 1\end{bmatrix}$	1 1

OR

Ans (iii)(b)	$X=\begin{bmatrix}x\\ y\end{bmatrix}=A^{-1}B=\frac{1}{3}\begin{bmatrix}1 & 1\\ -2 & 1\end{bmatrix}\begin{bmatrix}50\\ 550\end{bmatrix}=\frac{1}{3}\begin{bmatrix}600\\ 450\end{bmatrix}=\begin{bmatrix}200\\ 150\end{bmatrix}$ $\therefore \text{Area}=xy=200\times150=30,000$ sq m	1½ ½

29. Solution

Let the production manager produces x number of strips (of 10 tablets) of Paingo, y number of strips (of 10 tables) X -prene and z number of strips (of 10 tablets) Relaxo. ½

We have the following information from the question

	Paingo	X-prene	Relaxo	Availability
A	$2x$	$4y$	z	16000
C	$3x$	y	$2z$	10000
D	x	$3y$	$3z$	16000

From the table we have

$2x + 4y + z = 16000 \quad (1)$

$3x + y + 2z = 10000 \quad (2)$

$x + 3y + 3z = 16000 \quad (3)$

The matrices representation of above system of equations is

$$\begin{bmatrix}2 & 4 & 1\\ 3 & 1 & 2\\ 1 & 3 & 3\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}16000\\ 10000\\ 16000\end{bmatrix}$$ 1

or $AX = B$

Since, $\lvert A \rvert = 2(3-6) - 4(9-2) + 1(9-1)$ $= -6 - 28 + 8 = -26 \neq 0,$	½
Thus A^{-1} exists, so that the unique solution of $AX = B$ is $X = A^{-1}B$. Here, $\operatorname{adj} A = \begin{bmatrix} -3 & -9 & 7 \\ -7 & 5 & -1 \\ 8 & -2 & -10 \end{bmatrix}$	1
$A^{-1} = \frac{1}{\lvert A \rvert}(adj\ A) = \frac{1}{-26}\begin{bmatrix} -3 & -9 & 7 \\ -7 & 5 & -1 \\ 8 & -2 & -10 \end{bmatrix}$	½
Now, $X = A^{-1}B = \frac{1}{-26}\begin{bmatrix} -3 & -9 & 7 \\ -7 & 5 & -1 \\ 8 & -2 & -10 \end{bmatrix}\begin{bmatrix} 16000 \\ 10000 \\ 16000 \end{bmatrix} = \begin{bmatrix} 1000 \\ 3000 \\ 2000 \end{bmatrix}$ Hence, number of strips of Paingo, X -prene and Relaxo are 1000, 3000 and 2000 respectively	1½

30. Solution

Under equilibrium condition, For market A $82 - 3p_A + p_B = -5 + 15p_A \implies 18p_A - p_B = 87$	1½
For market B $92 + 2p_A - 4p_B = -6 + 32p_B \implies 2p_A - 36p_B = -98$	1
Now, $\lvert D \rvert = -646$, $\lvert D_A \rvert = -3230$, $\lvert D_B \rvert = -1938$	1 ½
$p_A = \frac{\lvert D_A \rvert}{\lvert D \rvert} = \frac{-3230}{-646} = 5$	½
$p_B = \frac{\lvert D_B \rvert}{\lvert D \rvert} = \frac{-1938}{-646} = 3$	½

31. Solution

$y = ax^2 + bx + c$

Owl passes through the points $(1,2), (2,1)$ and $(4,5)$. So, it must satisfy the given equation

Therefore,

$$\left.\begin{array}{l} 2 = a + b + c \\ 1 = 4a + 2b + c \\ 5 = 16a + 4b + c \end{array}\right\}$$ 1

Now, $D = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 16 & 4 & 1 \end{vmatrix} = 1(2-4) - 1(4-16) + 1(16-32) = -6 \neq 0$ ½

$D_a = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 5 & 4 & 1 \end{vmatrix} = 2(2-4) - 1(1-5) + 1(4-10) = -6$ ½

$D_b = \begin{vmatrix} 1 & 2 & 1 \\ 4 & 1 & 1 \\ 16 & 5 & 1 \end{vmatrix} = 1(1-5) - 2(4-16) + 1(20-16) = 24$ ½

and $D_c = \begin{vmatrix} 1 & 1 & 2 \\ 4 & 2 & 1 \\ 16 & 4 & 5 \end{vmatrix} = 1(10-4) - 1(20-16) + 2(16-32) = -30$ ½

$\therefore a = \frac{D_a}{D} = \frac{-6}{-6} = 1$; $,b = \frac{D_b}{D} = \frac{24}{-6} = -4$, $,c = \frac{D_c}{D} = \frac{-30}{-6} = 5$ 1½

Therefore, equation of the curve is $y = x^2 - 4x + 5$

When owl is sitting at $(0,k)$ then $x = 0 \Rightarrow k = 5$ ½

32. Solution

(i) $s(t)=at^2+bt+c\ ;t\geq 0$

Clearly, $(10,16),(20,22),(30,25)$ lie on the curve of $s(t)$.

Then, $100a+10b+c=16$

$400a+20b+c=22$

$900a+30b+c=25$ 1

(ii) Let, $A=\begin{pmatrix}100 & 10 & 1\\ 400 & 20 & 1\\ 900 & 30 & 1\end{pmatrix};X=\begin{pmatrix}a\\ b\\ c\end{pmatrix};B=\begin{pmatrix}16\\ 22\\ 25\end{pmatrix}$ ½

Then, the system becomes, $AX=B$

$|A|=100(-10)-400(-20)+900(-10)$

$=-1000+8000-9000$

$=-2000\neq 0$ ½

Now, $adjA=\begin{pmatrix}-10 & 500 & -6000\\ 20 & -800 & 6000\\ -10 & 300 & -2000\end{pmatrix}^T=\begin{pmatrix}-10 & 20 & -10\\ 500 & -800 & 300\\ -6000 & 6000 & -2000\end{pmatrix}$ 1

Therefore, $A^{-1}=\frac{1}{|A|}(adjA)=\frac{1}{-2000}\begin{pmatrix}-10 & 20 & -10\\ 500 & -800 & 300\\ -6000 & 6000 & -2000\end{pmatrix}$ ½

Then, $X=A^{-1}B=\frac{1}{-2000}\begin{pmatrix}-10 & 20 & -10\\ 500 & -800 & 300\\ -6000 & 6000 & -2000\end{pmatrix}\begin{pmatrix}16\\ 22\\ 25\end{pmatrix}$

$=\frac{1}{-2000}\begin{pmatrix}30\\ -2100\\ -14000\end{pmatrix}$

$=\begin{pmatrix}-\frac{3}{200}\\ \frac{21}{20}\\ 7\end{pmatrix}$ 1½

Therefore, $a=-\frac{3}{200},b=\frac{21}{20},c=7.$

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33. Solution

Q-35 x + y + z = 12
2x + 3y + 3z = 33
x – 2y + z = 0

$$\begin{bmatrix}1 & 1 & 1\\2 & 3 & 3\\1 & -2 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}12\\33\\0\end{bmatrix}$$ 1 Mark

$|A| = 3 \quad \neq 0$ ½ Mark

$$\text{adj}A = \begin{bmatrix}9 & -3 & 0\\1 & 0 & -1\\-7 & 3 & 1\end{bmatrix}$$ 2½ Mark

$$X = A^{-1}B \Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix} = \frac{1}{3}\begin{bmatrix}9 & -3 & 0\\1 & 0 & -1\\-7 & 3 & 1\end{bmatrix}\begin{bmatrix}12\\33\\0\end{bmatrix}$$

$$X = A^{-1}B \Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix} = \frac{1}{3}\begin{bmatrix}108-99+0\\12+0-0\\-84+99+0\end{bmatrix} = \frac{1}{3}\begin{bmatrix}9\\12\\15\end{bmatrix} = \begin{bmatrix}3\\4\\5\end{bmatrix}$$

Hence x = 3 , y = 4, z = 5 1 Mark

34. Solution

	$A = \begin{bmatrix}1 & 2 & 3\\3 & 2 & 2\\2 & 3 & 2\end{bmatrix}$ $\Rightarrow |A| = 9 \Rightarrow A^{-1}$ exists And $A^{-1} = \frac{1}{9}\begin{bmatrix}-2 & 5 & -2\\-2 & -4 & 7\\5 & 1 & -4\end{bmatrix}$	2
	$AX = B \Rightarrow X = A^{-1}B$ $\Rightarrow X = \frac{1}{9}\begin{bmatrix}-2 & 5 & -2\\-2 & -4 & 7\\5 & 1 & -4\end{bmatrix}\begin{bmatrix}85\\105\\110\end{bmatrix} = \begin{bmatrix}15\\20\\10\end{bmatrix}$ $\Rightarrow p_1 = 15, p_2 = 20, p_3 = 10$	3

CASE-BASED QUESTIONS

35. Solution

(iii) (a) $|A| = -40 + 20 = -20 \neq 0$ $(\frac{1}{2})$

adj (A) $= \begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix}$ **(1)**

$\Rightarrow A^{-1} = -\frac{1}{20}\begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix} = \frac{1}{20}\begin{bmatrix} 8 & -4 \\ 5 & -5 \end{bmatrix}$ $(\frac{1}{2})$

OR

(iii) (b) $X = \begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}B$

$= \frac{1}{20}\begin{bmatrix} 8 & -4 \\ 5 & -5 \end{bmatrix}\begin{bmatrix} 40 \\ -80 \end{bmatrix}$ $(\frac{1}{2})$

$= \frac{1}{20}\begin{bmatrix} 640 \\ 600 \end{bmatrix} = \begin{bmatrix} 32 \\ 30 \end{bmatrix}$ **(1)**

$\Rightarrow x = 32, y = 30$ $(\frac{1}{2})$

Sol.

(i) Let number of children be x and the amount donated to each child be ₹ y

$\therefore (x-8)(y+10) = xy$ and $(x+16)(y-10) = xy$ $(\frac{1}{2})$

i.e., $5x - 4y = 40$ and $5x - 8y = -80$ $(\frac{1}{2})$

(ii) $A = \begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$ **(1)**

36. Solution

Ans.

(a) $x + y + z = 10, 2x + y = 13, x + y = 4z$ **1**

(b) coefficient matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & -4 \end{bmatrix}$ **1**

(c) (i) Cofactor matrix of $A = \begin{bmatrix} -4 & 8 & 1 \\ 5 & -5 & 0 \\ -1 & 2 & -1 \end{bmatrix}$ **2**

OR

(ii) Adj (A) $= \begin{bmatrix} -4 & 5 & -1 \\ 8 & -5 & 2 \\ 1 & 0 & -1 \end{bmatrix}$ and $|A| = 5 \neq 0$

$\therefore A^{-1} = \frac{1}{5}\begin{bmatrix} -4 & 5 & -1 \\ 8 & -5 & 2 \\ 1 & 0 & -1 \end{bmatrix}$ **1**

Thus, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -4 & 5 & -1 \\ 8 & -5 & 2 \\ 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} 10 \\ 13 \\ 0 \end{bmatrix}$

$= \frac{1}{5}\begin{bmatrix} 25 \\ 15 \\ 10 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix}$

$\therefore$ 5 students in first group, 3 students in second group and 2 students in third group **1**