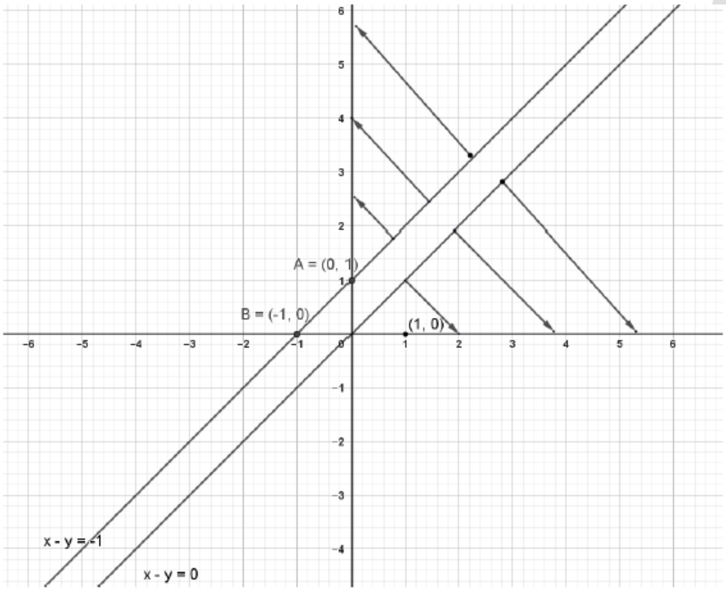
LINEAR PROGRAMMING PROBLEMS

1. (B) 140
2. (C) half-plane that neither contains the origin nor the points on the line **2x+3y=6**
3. (D) infinite
4. (A) I quadrant
5. (D) half plane that neither contains the origin nor the points on the line 3x + 2y = 6
6. (D) zero
7. (c) 31
8. (D) $2x + y \geq 10, x \geq 0, y \geq 0$
9. (A) $x + 2y \geq 3$
10. (a) $x + 2y \leq 5$; $x + y \leq 4$
11. (A) Feasible region
12. Find

Sol.



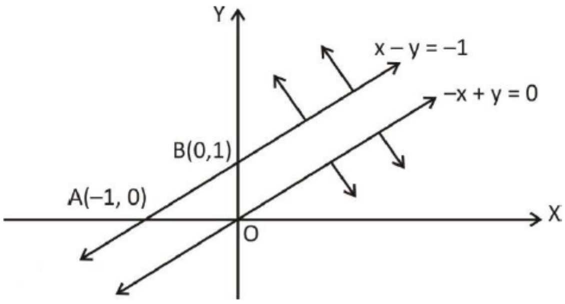
$(1\frac{1}{2})$ for correct graph

Since feasible region is empty, there is no solution to the problem. $(\frac{1}{2})$

13. Maximize

Solution:

The graph of the given constraints is



[1]

Here, the feasible region is empty.

So, there exists no solution to the given LPP. **[1]**

14. Two
Ans

Let us assume that tailor A works for x days and tailor B works for y days to complete the job.

Let Z denote the total labour cost.

The LPP for the given problem is:

Minimize $Z = 1500x + 2000y$ ½

subject to constraints,

$6x + 10y \geq 60$ or $3x + 5y \geq 30$ ½

$4x + 4y \geq 32$ or $x + y \geq 8$ ½

$x \geq 0, y \geq 0$ ½

15. The

16. A

Q-23 Let 'x' hectares and 'y' hectares of land be allocated to crop A and Crop B

Max Z = 8000x + 9500y. ½ Mark

Subject to $x + y \leq 10$; $2x + y \leq 50$; $x \geq 0$ *and* $y \geq 0$ 1½ mark

17. A

.

Let the number of hardcopy and paperback copies be x and y respectively 1

$\Rightarrow$ Maximum profit Z = (72x + 40y) —(9600 + 56x + 28y) = 16x + 12y— 9600

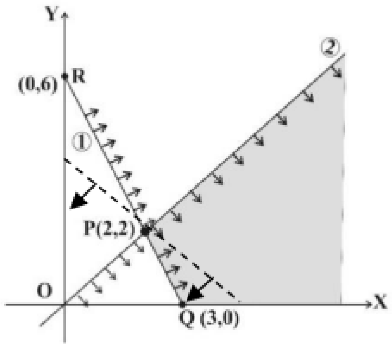
Subject to constraints: 1

$x + y \leq 960$

$5x + y \leq 2400$

$x, y \geq 0$

18. The



Corner Point	Z=3x+2y
P (2, 2)	10
Q (3, 0)	9

1

The smallest value of Z is 9. Since the feasible region is unbounded, we draw the graph of $3x + 2y < 9$. The resulting open half plane has points common with feasible region, therefore Z = 9 is not the minimum value of Z. Hence the optimal solution does not exist. 1

19. There

Sol. Let x kg of nitrogen and y kg of phosphoric acid is used for minimum cost.

$\therefore$ the objective function is

Minimize $Z = 6x + 5y$ ½

Subject to the constraints $10\% \times x + 5\% \times y \geq 14$ or $2x + y \geq 280$ 1

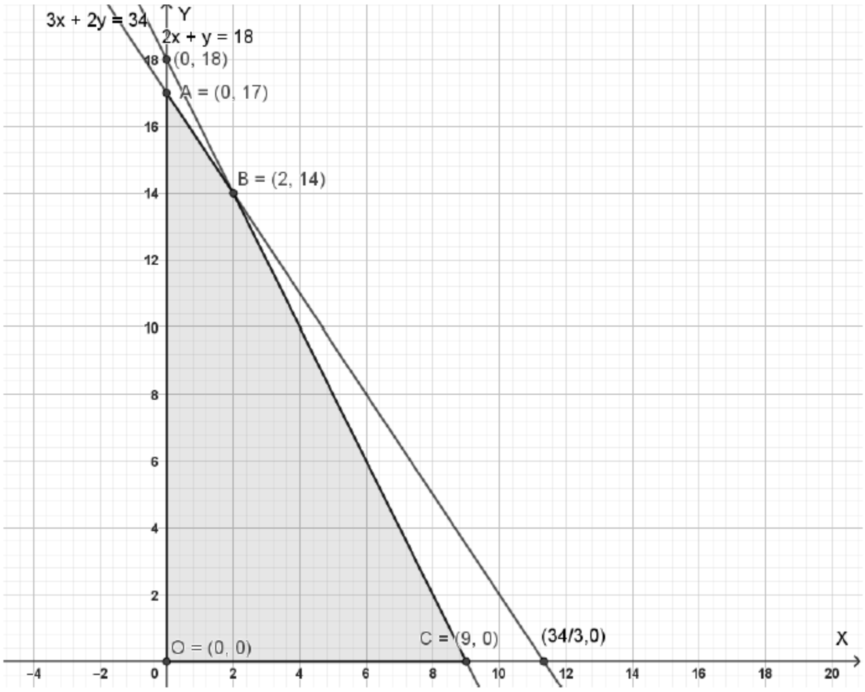
and $6\% \times x + 10\% \times y \geq 14$ or $3x + 5y \geq 700$ 1

$x, y \geq 0$ ½

Note: * Marks should be awarded for the formation of equations $2x + y = 280$ and $3x + 5y = 700$ instead of inequations in Hindi medium only.

20. Solve

Sol.



1½ for correct graph

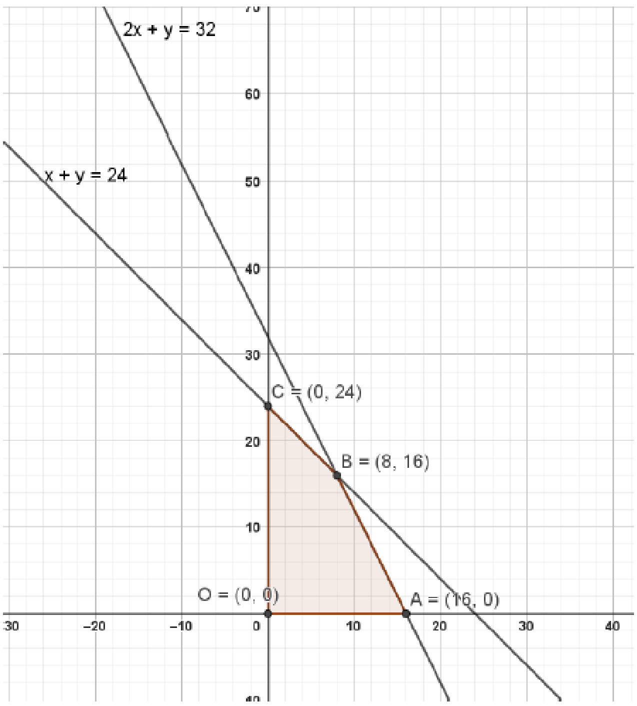
Corner Points	Value of Z
O (0,0)	0
A (0, 17)	510
B (2,14)	520
C (9, 0)	450

1 for correct table

∴ Maximum Z = 520 at B (2, 14) ½

21. Maximise

Sol.



(2)

Corner Points	Value of Z
O (0,0)	0
A (16,0)	4800
B (8,16)	5440 → Max Value
C (0,24)	4560

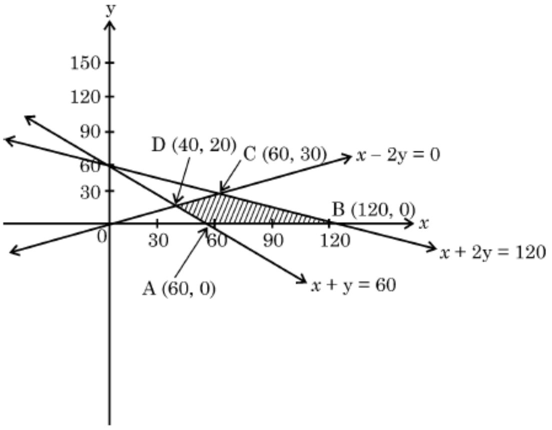
(1)

So Z is maximum at B (8,16)

Max Value of Z = 5440

22. Minimise

Sol.



1½ for correct graph

Corner Points	Z = 5x+10y
A (60, 0)	300
B (120, 0)	600
C (60, 30)	600
D (40, 20)	400

1 for correct table

Z is minimum at A(60, 0) ½

23. Formulate

Ans. Let $\boldsymbol{x}$ and $\boldsymbol{y}$ denote the number of rings and chains respectively

Maximize $\boldsymbol{Z = 300\,x + 190\,y}$ **1**

Subject to constraints

$$\boldsymbol{x + y \le 24}$$
$$\boldsymbol{x + \frac{y}{2} \le 16}$$
$$\boldsymbol{x, y \ge 0}$$

2

24. A

Let x be the number of units of Product A produced per day

and y be the number of units of Product B produced per day ½

The objective is to **maximize the profit**, which is given by:

$$Z = 30\,x + 40\,y$$

subject to the following constraints: **1**

$$4\,x + 6\,y \le 500$$
$$x \le 80$$
$$y \le 60$$
$$x \ge 0, y \ge 0$$

1½

25. A

Let the number of necklaces manufactured be x, and the number of bracelets manufactured be y.

According to question,

$x+y\le 25$ and

$\frac{x}{2}+y\le 14$

The profit on one necklace is ₹ **100** and the profit on one bracelet is ₹ **300**.

Let the profit (the objective function) be Z, which has to be maximized.

Therefore, required LPP is

Maximize $Z=100x+300y$ **1**

Subject to the constraints

$x+y\le 25$ **½**

$\frac{x}{2}+y\le 14$ **1**

$x, y\ge 0$ **½**

26. A

Ans Let x packets of food P and y packets of food Q be mixed. The LPP is:

Minimize $Z=6x+3y$ ½

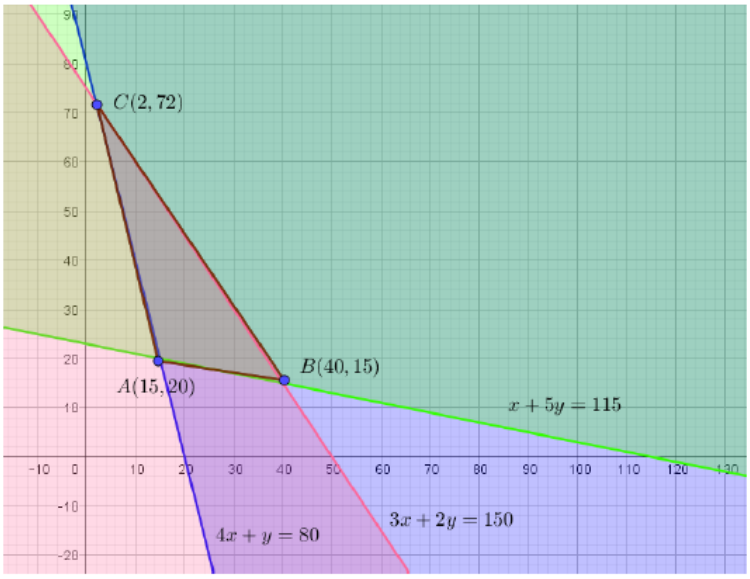
subject to constraints

$12x+3y\ge 240$ or $4x+y\ge 80$

$4x+20y\ge 460$ or $x+5y\ge 115$ 1½

$6x+4y\le 300$ or $3x+2y\le 150$

$x\ge 0, y\ge 0$



2 marks for the correct graph.

ABC forms the feasible bounded region, where $A(15, 20)$, $B(40,15)$ and $C(2,72)$.

Now, $Z_A = 150, Z_B = 285, Z_C = 228$

Z is minimum at $A(15, 20)$. 1

$Z_{\min} = 150$, when 15 packets of food P and 20 packets of food Q are mixed.

27. A

Ans.

Types of boxes	Thickness (in cm)	Weight (in kg)
Type 1	6	1
Type 2	4	$1\frac{1}{2}$
Max Availability	96 cm	21 kg

Let the two types of boxes be x and y respectively

Let Z denote the maximum number of books that can be accommodated in the shelf

LPP is

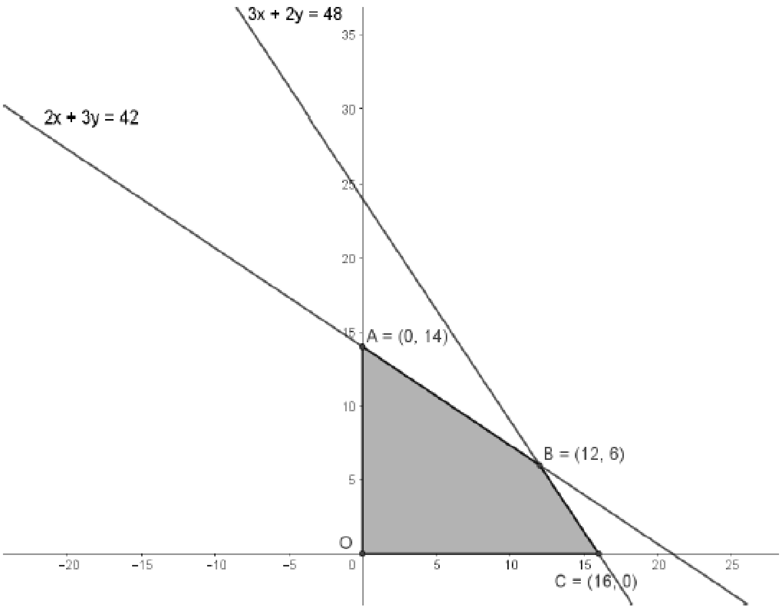
$\therefore Z = x + y$ **1**

Subject to constraints

$6x + 4y \leq 96$ or $3x + 2y \leq 48$

$x + \frac{3}{2}y \leq 21$ or $2x + 3y \leq 42$ **1½**

$x, y \geq 0$

	1½ For graph with correct region
Here, $(Z)_A = 14, (Z)_B = 18, (Z)_C = 16$ So, Z is maximum at B Hence, the shelf should be filled with 12 books of type 1 and 6 books of type 2	1

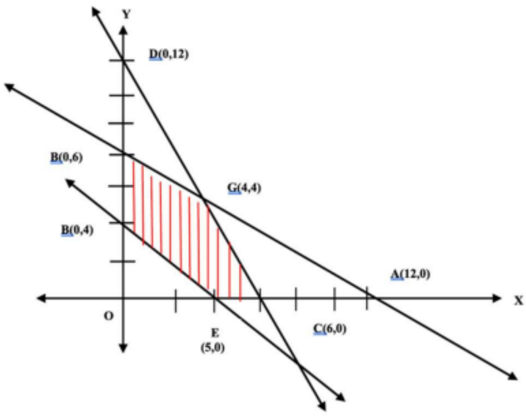
28. A

Q-33 Let 'x' and 'y' be the number of units of items M and N respectively.

We have : $x \geq 0, y \geq 0$

$x + 2y \leq 12 ; 2x + y \leq 12 ; \; x + \frac{5}{4}y \geq 5.$ 1½ Mark

Max Z = 600x + 400 y 1 Mark



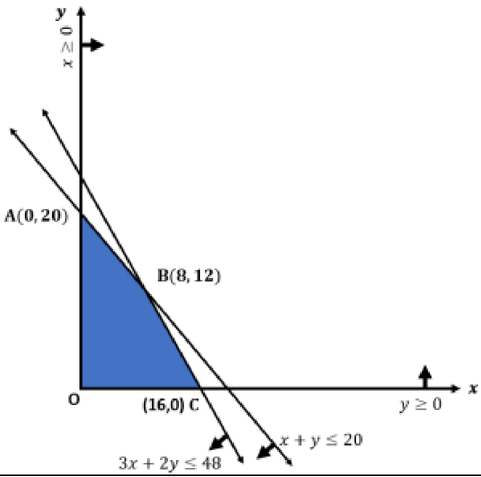
Graph 1½ Mark

Corner Point	Z = 600x + 400y
E : (5,0)	3000
C : (6,0)	3600
G : (4,4)	**4000 (Maximum)**
B : (0,6)	2400
F : (0,4)	1600

Hence maximum profit is ₹4000 when 4 units of each item M and N are produced. 1 Mark

29. Rahul is

Let the number of tables and chairs be x and y respectively
(Max profit) Z = $22x + 18y$
Subject to constraints:
$x + y \leq 20$
$3x + 2y \leq 48$
$x, y \geq 0$ 1.5



2

The feasible region OABCA is closed (bounded)

Corner points	Z = 22 x + 18 y
O (0,0)	0
A (0,20)	360
B (8,12)	392
C (16,0)	352

Buying 8 tables and 12 chairs will maximise the profit 1.5

CASE-BASED QUESTIONS

30. There

Sol.

(i) P to C $= 9 - (x + y)$ **1**

(ii) Q to A $= (4 - x)$

Q to B $= (4 - y)$ **½**

Q to C $= 6 - [9 - x - y] = (x + y - 3)$ **½**

(iii) $Z = 160\,x + 100\,y + 150\,(9 - x - y) + 100\,(4 - x) + 120(4 - y) + 100(x + y - 3)$ **1**

$= 10x - 70y + 1930$ **1**

OR

(iii) $x + y \leq 9$ **½**

$x + y \geq 3$ **½**

$x \leq 4, y \leq 4$

$x, y \geq 0$ **1**

31. In

Sol.

(i) $x \geq 0, y \geq 0$ **(1)**

(ii) (a) $2x - y \leq 0,$ **(1)**

$2x + y \leq 200$ **(1)**

OR

(ii) (b) $2x + y \leq 200,$ **(1)**

$2x - y \geq 0$ **(1)**

(iii) Corner points of R_1 are $A(0,50), B(20,40), C(50,100)$ and $D(0,200)$

$\mathbf{Z_A = 100;\ Z_B = 180; Z_C = 450;\ Z_D = 400}$ **$(\frac{1}{2})$**

So, Z is maximum at C and maximum value of $Z = 450$ **$(\frac{1}{2})$**

32. A factory

Sol.

(i) $Z = 10x + 20y$ **(1)**

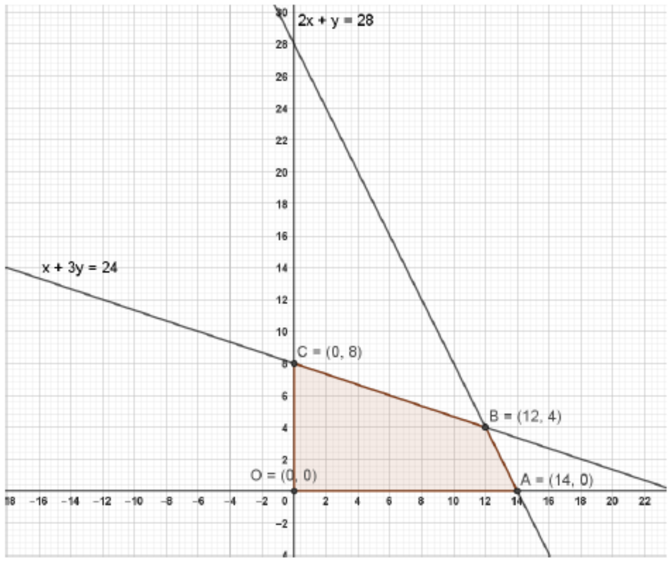
(ii) $x + 3y \leq 24$ **(1)**

(iii) (a) other constraints are

$2x + y \leq 28$

$x \geq 0$

$y \geq 0$



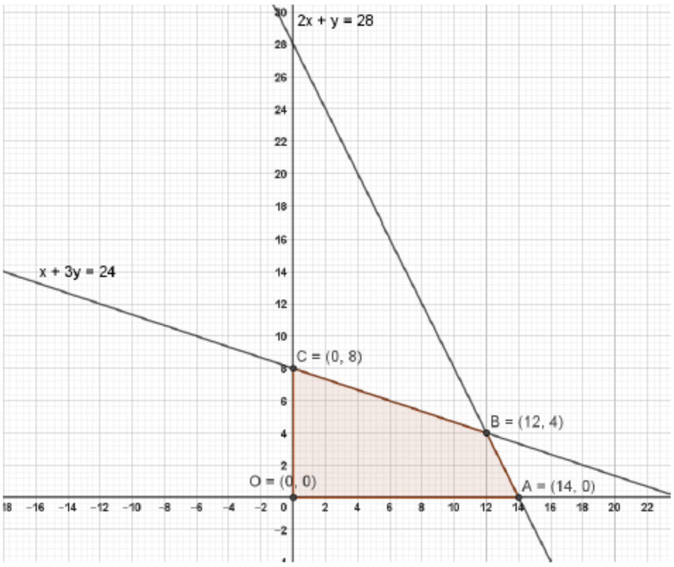
(1)

Corner Points	Value of Z
O (0,0)	0
A (14,0)	140
B (12,4)	200 → Max value
C (0,8)	160

(1)

∴ P is maximum at B (12,4); which is ₹ 200

(iii) (b)



(1)

Corner Points	Value of Z
O (0,0)	0
A (14,0)	140
B (12,4)	200 → Max value
C (0,8)	160

(1)

12 bats and 4 rackets

33. A

Solution:

Let the mixture contain x kg of food F_1 and y kg of food F_2. Then, LPP becomes

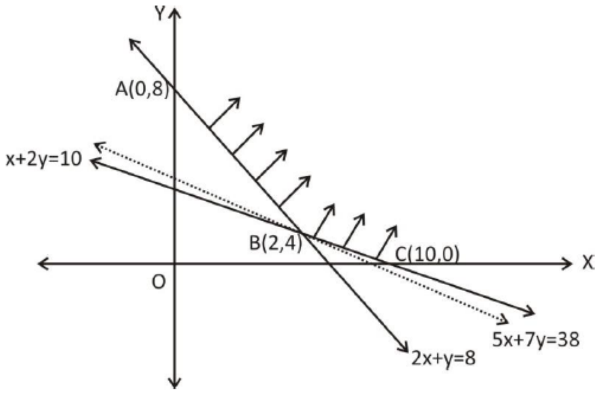
(a) Minimize $Z = 5x + 7y$ $\left[\frac{1}{2}\right]$

Subject to the constraints

$2x + y \geq 8$

$x + 2y \geq 10$

$x \geq 0, y \geq 0$ **[1]**



(b) The corner points are A(0, 8), B(2,4) and C (10, 0). **[1]**

The value of Z at these corner points are

$Z_A = 56$; $Z_B = 38$; $Z_C = 50$ $\left[\frac{1}{2}\right]$

Since the feasible region is unbounded, we draw the graph of

$5x + 7y < 38$.

As the graph of $5x + 7y < 38$ does not have any point common with the Feasible region, so the minimum cost of the mixture is ₹ 38. **[1]**

34. The

Sol. (i) $x + 2y = 12$ **1**

(ii) $4x + 5y = 20$ **1**

(iii) (a)

$x + 2y \le 12, \quad 4x + 5y \ge 20,$ ½

$2x + y \le 12,$ **1**

$x \ge 0, y \ge 0$ ½

OR

(iii) (b)

Corner Points	Value of Z
B (6,0)	3600
X (4,4)	4000
C (0,6)	2400
E (0,4)	1600
F (5,0)	3000

1½

Maximum value of Z = 4000 ½

35. A diet

36. If a

(i) Let the distance the man travels at 25 km/hr be denoted by x and the distance he travels at 40 km/hr be denoted by y **½**

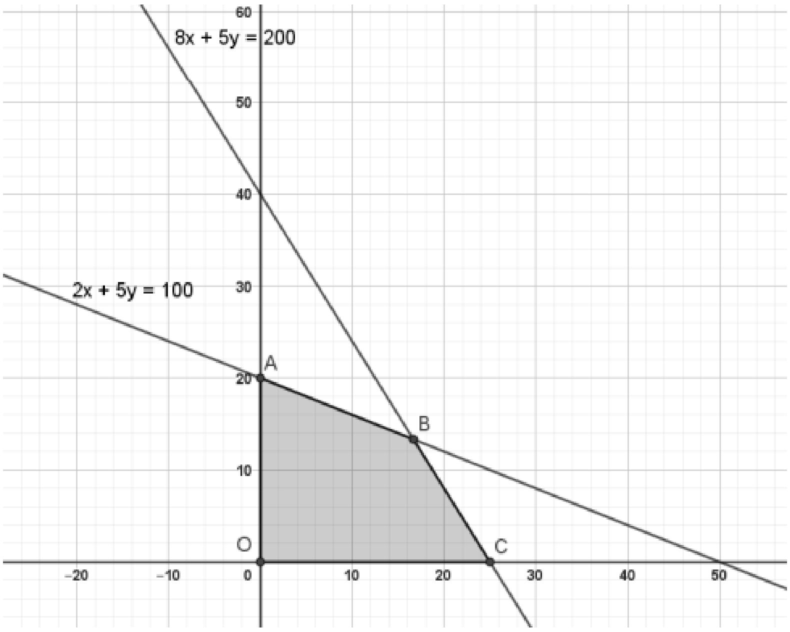
Linear programming problem is

Objective function is to Maximize $Z = x + y$ **½**

Subject to the constraints:

$$\left.\begin{array}{l} \frac{x}{25} + \frac{y}{40} \le 1 \text{ i.e., } 8x + 5y \le 200 \\ 2x + 5y \le 100 \\ x \ge 0, y \ge 0 \end{array}\right\}$$ **1**

(ii)



Corner Points	Value of Z
$O(0,0)$	0
$A(0,20)$	20
$B\left(\frac{50}{3},\frac{40}{3}\right)$	30
$C(25,0)$	25

1½

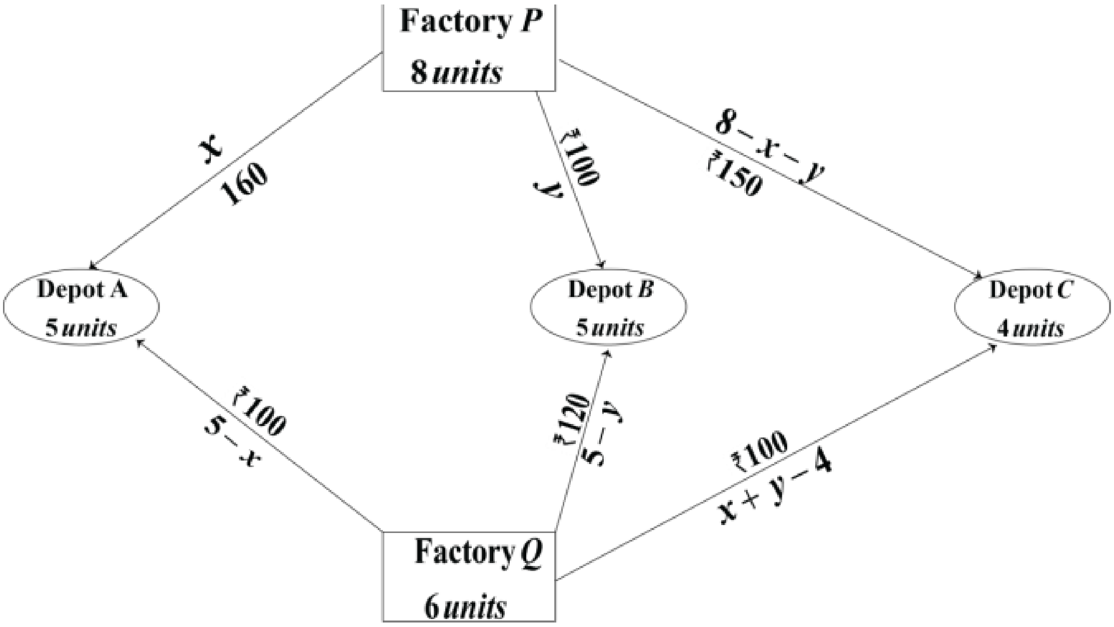
Thus, the maximum distance the man can travel within one hour is **30 km.** ½

37. A

(i) Let the factory P supply x units per week to depot **A** and y units to depot **B**

so that it supplies $8-x-y$ units to depot **C**. Obviously $0 \le x \le 5, 0 \le y \le 5, 0 \le 8-x-y \le 4.$

The given data can be represented diagrammatically as:



Thus, total transportation cost (in ₹)

$$=160x+100y+150(8-x-y)+100(5-x)+120(5-y)+100(x+y-4)=10(x-7y+190).$$

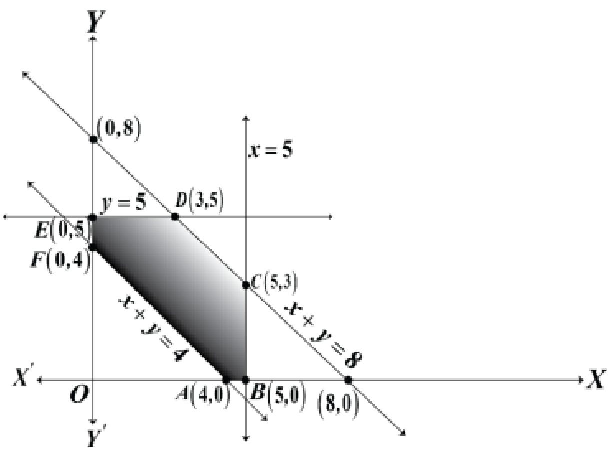
Hence the given problem can be formulated as an L.P.P as: 1

Minimize $Z=10(x-7y+190)$

subject to the constraints

$$\left.\begin{aligned} &x+y\geq 4,\\ &x+y\leq 8,\\ &x\leq 5,\\ &y\leq 5\\ &x\geq 0, y\geq 0\end{aligned}\right\}$$ 1

(ii) The feasible region corresponding to these in equations is shown shaded in the figure given below.



1

Corner Points	Value of $Z=10(x-7y+190)$
A (4,0)	1940
B (5,0)	1950
C (5,3)	1740
D (3,5)	1580
E (0,5)	**1550 $\rightarrow$ Minimum**
F (0,3)	1690

1

We observe that Z is minimum at point $E(0, 5)$ and minimum value is ₹ 1550.

Hence $x=0,\ y=5$. Thus for minimum transportation cost, factory P should supply $0,\ 5,\ 3$ units to depots **A, B, C** respectively and factory **Q** should supply $5,\ 0,\ 1$ units respectively to depots **A, B, C**.

38. S & D

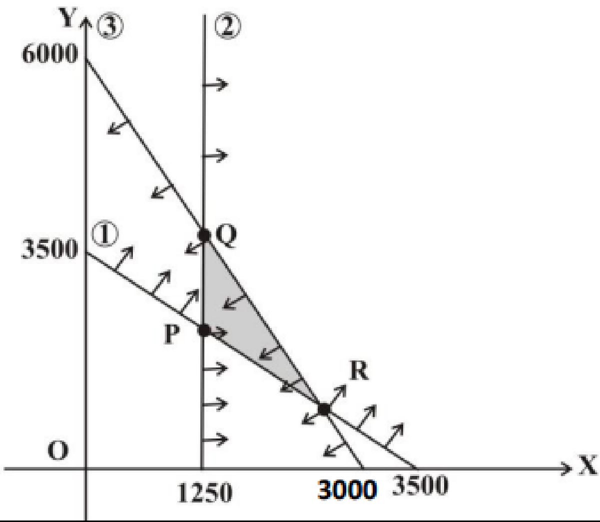
Let the company produces x and y gallons of alkaline solution and base oil respectively, also let C be the production cost.

Min $C = 200x + 300y$

subject to constraints:

$$x + y \geq 3500 \ldots .(1)$$
$$x \geq 1250 \quad \ldots .(2)$$
$$2x + y \leq 6000 \ldots (3)$$
$$x, y \geq 0$$

$1\frac{1}{2}$



$1\frac{1}{2}$

Corner Points	$C = 200x + 300y$
P(1250, 2250)	₹9,25,000

Q(1250, 3500)	₹13,00,000
R(2500, 1000)	₹8,00,000

Minimum cost is 8,00,000 when 2500 gallons of alkaline solutions & 1000 gallons of base oil are manufactured. 1