PROBABILITY

1. (C) $\frac{1}{3}$
2. (B) $\frac{1}{5}$
3. (A) 1
4. C) k + 1
5. (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is the not the correct explanation of the Assertion (A).
6. (B) $\frac{1}{e}$
7. (C) 2
8. (D) $\frac{2}{3}$
9. (C) $\frac{2}{e^2}$
10. (C) Mean
11. (A) 0.7
12. (D) $\frac{51}{101}$
13. (C) $\frac{48}{5^5}$
14. (A) e^{-m}
15. (c) mean
16. (C) 2
17. Assertion (A) is false and Reason (R) is true
18. (c) 0.5
19. (B) 3.8
20. (A) $\frac{4}{3}$
21. (D) 32
22. (A) Both A and R are true and R is the correct explanation of A
23. (A) Assertion is correct and Reason is the correct explanation for it
24. (C) $\frac{3}{7}$
25. (D) 1
26. (B) $1-\left(\frac{19}{20}\right)^6$
27. (A) -50
28. (d) 3
29. C) $\frac{11}{1024}$
30. (A) 0.1587
31. (b) 93.32%
32. (d) 0.8647
33. (c) 0.9876

34. If

Sol.	Here, $n=6, p=\frac{1}{2}, q=\frac{1}{2}$	½
	P (at least 4 heads in 6 throws) $= P(X \geq 4)$	
	$= 6_{C_4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^2 + 6_{C_5}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^1 + 6_{C_6}\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^0$	**1**
	$= 22\left(\frac{1}{2}\right)^6 = \frac{22}{64}\ or\ \frac{11}{32}$	½

35. Given

Sol. (i) $Z = \frac{X-\mu}{\sigma} = \frac{15-9}{3} = 2$ **1**

(ii) $4 = \frac{X-9}{3} \Rightarrow X = 21$ **1**

36. Suppose

Sol. Let p be the probability that an item is defective so, $p = \frac{2}{100} = 0\cdot02$. $(\frac{1}{2})$

Here $n = 100 \therefore m = np = 2$ $(\frac{1}{2})$

$P(X = r) = \frac{m^r}{r!}e^{-m} = \frac{2^r e^{-2}}{r!}$ $(\frac{1}{2})$

$\Rightarrow \mathbf{P(X = 3) = \frac{2^3 e^{-2}}{3!} = \frac{4}{3} \times 0\cdot135 = 0\cdot18}$ $(\frac{1}{2})$

37. Find t

Sol. X can take the values 0, 1 and 2

$p = \frac{2}{6} = \frac{1}{3}, q = \frac{2}{3}$ ½

X	0	1	2
P(X)	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

1½

38. If

Ans For $X = 75$, $Z = \frac{75-70}{5} \Rightarrow Z = 1$ ½

$P(X > 75) = P(Z > 1)$

$= 0.5 - P(0 < Z < 1)$ 1

$= 0.5 - 0.3413$

$= 0.1587$ ½

39. If

Ans Let X be a poisson variate with parameter λ.

Then $P(X = r) = \frac{e^{-\lambda}\lambda^r}{r!}$

Here $P(X = 0) = e^{-\lambda}$ and $P(X = 1) = \lambda e^{-\lambda}$ 1

$\therefore P(X = 0) = P(X = 1) = \alpha \Rightarrow e^{-\lambda} = \lambda e^{-\lambda} = \alpha$

$\Rightarrow \lambda = 1$ ½

$\Rightarrow \alpha = \frac{1}{e}$ i.e. $\alpha = e^{-1}$ ½

40. It

	Then $p = \frac{2}{100}$. Here $n = 100$ So, $m = np = 2$ Let X denote the number of defective screws in a packet of 100 screws. Then X follows the probability distribution as $P(X = r) = e^{-m}\frac{m^r}{r!}, r = 0,1,2 \ldots$ P(no defective screw) $= P(X = 0) = e^{-m}\frac{m^0}{0!} = e^{-2} = 0.14$	1 1

41. If

Ans.	Let X follows Poisson distribution with parameter m Then Variance $= m = (\sqrt{3})^2 = 3$ $\therefore P(X = r) = e^{-m}\frac{m^r}{r!} = e^{-3}\frac{3^r}{r!}, r = 0,1,2 \ldots$ Hence $P(X > 0) = 1 - P(X = 0) = 1 - e^{-3} = 1 - 0.05 = 0.95$	½ 1½

42. A

$P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) = 1$ $\Rightarrow 1 \times a + 2 \times a + 3 \times a + b = 1$ $\Rightarrow 6\,a + b = 1 \ldots$ (i) $E(X) = 1 \times a + 2 \times 2a + 3 \times 3a + 4 \times b = 2.8$ $\Rightarrow 14\,a + 4\,b = 2.8 \ldots$ (ii) Solving (i) and (ii), we get $a = 0.12$ and $b = 0.28$	½ ½ ½+½

43. A

Here, $\lambda = 2$ Required probability $= P(X = 3/X \geq 1) = \frac{P(X=3 \cap X \geq 1)}{P(X \geq 1)} = \frac{P(X=3)}{P(X \geq 1)}$ $P(X = 3) = \frac{2^3 e^{-2}}{3!} = 0.19$ $P(X \geq 1) = 1 - P(X = 0) = 1 - \frac{2^0 e^{-2}}{0!} = 1 - 0.14 = 0.86$ $\therefore P(X = 3/X \geq 1) = \frac{0.19}{0.86} = 0.22$	½ ½ ½ ½

44. A

$\lambda = \frac{2}{1000} \times 5000 = 10$ P(more than 1 failure) $= P(X > 1) = 1 - [P(X = 0) + P(X = 1)]$ $= 1 - \left[\frac{10^0 e^{-10}}{0!} + \frac{10^1 e^{-10}}{1!}\right]$ $= 1 - e^{-10}(1 + 10)$ $= 1 - 4.54 \times 10^{-5} \times 11$ $= 1 - 0.0004994 = 0.9995$ approx.	½ ½ ½ ½

45. The

Given, $p=\frac{20}{100}=\frac{1}{5}\Rightarrow q=\frac{4}{5}$ and $n=6$ ½

Now, $P(X=x)={}^6C_x\left(\frac{1}{5}\right)^x\left(\frac{4}{5}\right)^{6-x}$, $x=0,1,....,6$

So, the required probability that out of six workers **4** or more will catch the disease is

$$P(X\geq 4)=P(X=4)+P(X=5)+P(X=6)$$
$$={}^6C_4\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^2+{}^6C_5\left(\frac{1}{5}\right)^5\left(\frac{4}{5}\right)^1+{}^6C_6\left(\frac{1}{5}\right)^6\left(\frac{4}{5}\right)^0$$ 1
$$=\frac{265}{5^6}\text{ or } 0.017 .$$ ½

46. The

We have, mean $\mu=12$ and standard deviation $\sigma=2$, i.e., $X\sim N(\mu,\sigma^2)$

(i) Let X denote the count of the months for which this machine lasts.

The probability of an item produced by this machine will last less than 7 months is

$P(X<7)$

For $X=7$, $Z=\frac{7-12}{2}=-\frac{5}{2}$ ½

Now,

$$P(X<7)=P\left(Z<-\frac{5}{2}\right)=P\left(Z>\frac{5}{2}\right)$$
$$=1-P\left(Z<\frac{5}{2}\right)=1-0.9938=0.0062$$ ½

(ii) The probability of an item produced by this machine will last more than 7 months and less than 14 months is $P(7<X<14)$

For $X=7$, $Z=\frac{7-12}{2}=-\frac{5}{2}$

and for $X=14$, $Z=\frac{14-12}{2}=1$ ½

$$P(7<X<14)=P\left(-\frac{5}{2}<Z<1\right)$$
$$=P(Z<1)-P\left(Z<-\frac{5}{2}\right)$$
$$=0.8413-0.0062=0.8351$$ ½

47. Two

Sol. Numbers on the dice are 1, 2, 3, 4, 5, 6

$\therefore$ X can take the values 1, 2, 3, 4, 5 ½

X	1	2	3	4	5
P(X)	$\frac{5}{15}$	$\frac{4}{15}$	$\frac{3}{15}$	$\frac{2}{15}$	$\frac{1}{15}$
X P(X)	$\frac{5}{15}$	$\frac{8}{15}$	$\frac{9}{15}$	$\frac{8}{15}$	$\frac{5}{15}$

1

1

$E(X) = \sum X\,P(X) = \frac{35}{15} = \frac{7}{3}$ ½

48. If

Sol. Mean $= np = \frac{4}{3}$, variance $= npq = \frac{8}{9}$ ½

$\Rightarrow q = \frac{8}{9} \times \frac{3}{4} = \frac{2}{3}$ ½

$\therefore p = 1 - \frac{2}{3} = \frac{1}{3}$ ½

$n \times \frac{1}{3} = \frac{4}{3} \Rightarrow n = 4$ ½

$P(x = 1) = 4_{C_1} \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^3 = 4 \times \frac{1}{3} \times \frac{8}{27}$ ½

$= \frac{32}{81}$ ½

49. The

Sol. Given, $p = 0.007, n = 400$ ½

$\therefore\ \lambda = np = 400 \times 0.007 = 2.8$ 1

Now, $P(X = 2) = \frac{(2.8)^2 e^{-2.8}}{2!}$ 1

$= \frac{7.84}{2} \times 0.0608 = 0.2383$ ½

50. Given

Sol. Here, $\mathbf{Z} = \frac{\mathbf{X-100}}{\mathbf{10}}$ $\left(\frac{1}{2}\right)$

$\mathbf{P(90 < X < 110) = P(X < 110) - P(X < 90)}$

$\mathbf{\Rightarrow P(90 < X < 110) = P(-1 < Z < 1) = P(Z < 1) - P(Z < -1)}$ (2)

$\mathbf{= 0.8413 - 0.1587}$

$\mathbf{= 0.6826}$ $\left(\frac{1}{2}\right)$

51. An

Sol. Let A be the event of obtaining two sixes in the first five throws of a die. Let B be the event of obtaining a six in the sixth throw of a die.

Then required probability = P(AB) = P(A) P(B)

Here, P(B) = $\frac{1}{6}$ and P(A) = $5_{C_2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3 = \frac{625}{3888}$ **(2)**

Thus, Required probability = $\frac{625}{3888} \times \frac{1}{6} = \frac{625}{23328}$ **(1)**

52. A

Ans.

Repeated tosses of a fair coin qualify as Bernoulii's trials

Let X denote the number of trails in an experiment of such trials and hence is the binomial distribution

Here $n = 9, p = \frac{1}{2}$ and $q = 1 - \frac{1}{2} = \frac{1}{2}$

(a) $P(\text{exactly 5 success}) = P(X = 5) = 9_{C_5}p^5q^4 = 9_{C_5}\left(\frac{1}{2}\right)^9$

$= \frac{63}{256}$ **1**

(b) $P(\text{at least 5 successes}) = P(X \geq 5)$

$= \left(\frac{1}{2}\right)^9\left[9_{C_5} + 9_{C_6} + 9_{C_7} + 9_{C_8} + 9_{C_9}\right]$

$= \frac{256}{512} = \frac{1}{2}$ **1**

(c) $P(\text{at most 5 successes}) = P(X \leq 5) = 1 - P(X > 5)$

$= 1 - \left(\frac{1}{2}\right)^9\left[9_{C_6} + 9_{C_7} + 9_{C_8} + 9_{C_9}\right]$

$= 1 - \frac{130}{512} = \frac{382}{512} = \frac{191}{256}$ **1**

53. Let

Ans.

x_i	0	1	2	3	otherwise
$P(x_i) = p_i$	0.2	k	$2k$	$2k$	0

½

(i) As $\sum p_i = 1$, we have

$0.2 + k + 2k + 2k = 1$

$\Rightarrow 5k = 0.8 \Rightarrow k = 0.16$ **1**

(ii) $P(X = 2) = 2k = 0.32$ ½

$P(X \geq 2) = 2k + 2k = 0.64$ ½

$P(X \leq 2) = 0.2 + k + 2k = 0.68$ ½

54. 2000

Ans.

Let X denote the marks of the student.

Mean $(\mu) = 30$, S.D $(\sigma) = 6.25$

$$Z = \frac{X-\mu}{\sigma} = \frac{X-30}{6.25}$$

(i) When $X = 20, Z = -1.60$ ½

When $X = 40, Z = 1.60$ ½

$\therefore P(20 \le X \le 40) = P(-1.60 \le Z \le 1.60)$

$= P(-1.60 \le Z \le 0) + P(0 \le Z \le 1.60)$

$= 2P(0 \le Z \le 1.60)$

$= 2 \times 0.4452 = 0.8904$

Thus, out of 2000 students, the expected number of students getting marks between 20 and 40 $= 2000 \times 0.8904 = 1780.8$ or 1781 1

(ii) When $X = 25, Z = -0.80$

So, $P(X < 25) = P(Z < -0.8) = P(Z > 0.8)$

$= P(Z \ge 0) - P(0 \le Z \le 0.8)$

$= 0.5 - 0.2881 = 0.2119$

Thus, out of 2000 students the expected number of students getting marks less than 25 $= 2000 \times 0.2119 = 423.8$ or 424 1

55. A

Given, mean $= \lambda = 3.2$ ½

Let X be the number of bicycle riders which use the cycle track.

Required probability $= P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$$= \frac{e^{-3.2}(3.2)^0}{0!} + \frac{e^{-3.2}(3.2)^1}{1!} + \frac{e^{-3.2}(3.2)^2}{2!}$$ 1½

$= e^{-3.2}(1 + 3.2 + 5.12)$

$= 0.041 \times 9.32 = 0.618$ ½

Also, mean expectation = variance of $X = \lambda = 3.2$ ½

56. An o

(i) **We have,** $\sum_{i=1}^{8} P(X = i) = 1$

$$\Rightarrow p+2p+2p+p+2p+p^2+2p^2+7p^2+p=1$$ ½

$$\Rightarrow 10p^2+9p-1=0$$

$$\Rightarrow (10p-1)(p+1)=0$$
$$\Rightarrow p\neq -1$$
$$\therefore p=\frac{1}{10}$$ 1

(ii)

$$\text{Mean, } E(X)=\sum_{i=1}^{8} i\,P(X=i)$$ ½

$$=1\times p+2\times p+3\times 2p+4\times p+5\times 2p+6\times p^2+7\times 2p^2+8\times(7p^2+p)$$ ½

$$=33p+76p^2$$

$$=\frac{33}{10}+\frac{76}{100}=\frac{203}{50}$$ ½

57. If

We have, $p=0.01=\frac{1}{100}\Rightarrow q=\frac{99}{100}$ ½

Let number of Bernoulli trials be n.

Now, the binomial distribution formula is for any random variable (X) is given by

$$P(X=x)={}^{n}C_x\left(\frac{1}{100}\right)^x\left(\frac{99}{100}\right)^{n-x}$$

So, the probability of at least one success is

$$P(X\geq 1)=1-P(X=0)=1-{}^{n}C_0\left(\frac{1}{100}\right)^0\left(\frac{99}{100}\right)^n=1-\left(\frac{99}{100}\right)^n$$ 1

According to condition, $P(X\geq 1)\geq 0.5\Rightarrow 1-\left(\frac{99}{100}\right)^n\geq 0.5\Rightarrow\left(\frac{99}{100}\right)^n\leq 0.5$ ½

$$\Rightarrow n\log_{10}\frac{99}{100}\leq\log_{10}0.5\Rightarrow n\geq\frac{\log_{10}0.5}{\log_{10}0.99};\quad(\text{as }\log_{10}0.99<0)$$ ½

$\left[\text{Using }\log_{10}2=0.3010\text{ and }\log_{10}99=1.9956\right]\ \Rightarrow n\geq 68.409\Rightarrow n=69\ [\because n\in\mathbb{N}]$. ½

58. LX

Sol.	(i) $0.1 + k + 2k + 2k + k = 1$	**(1)**
	$\Rightarrow 0.1 + 6k = 1$	
	$\Rightarrow k = \frac{3}{20}$	**(1)**
	(ii) $P(X \geq 2) = P(2) + P(3) + P(4)$	$(\frac{1}{2})$
	$= 2k + 2k + k$	
	$= 5k = \frac{3}{4}$	**(1)**
	(iii) $P(X \leq 2) = P(0) + P(1) + P(2)$	$(\frac{1}{2})$
	$= 0.1 + k + 2k$	
	$= \frac{1}{10} + \frac{9}{20} = \frac{11}{20}$	**(1)**

59. A

Sol.	Here, mean expectation $= \lambda = 2$	**(1)**
	$P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$	
	$P(X \leq 3) = P(0) + P(1) + P(2) + P(3)$	**(1)**
	$= e^{-2}\left(1 + 2 + 2 + \frac{4}{3}\right)$	**(2)**
	$= 0.13534 \times \frac{19}{3} = 0.8571 \text{ or } 0.86$	**(1)**

60. An

Ans	Let A be the event of obtaining two sixes in the first five throws of a die and B be the event of obtaining a six in the sixth throw of a die.	
	Then, required probability = $P(AB) = P(A).P(B)$	
	Let p denotes the probabilty of success i.e. getting a six in a single throw of a die. Then, $p = \frac{1}{6}$, $q = 1 - p = \frac{5}{6}$	1
	$\therefore P(A) = {}^5C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3 = \frac{625}{3888}$	2
	Also, $P(B) = \frac{1}{6}$	1
	Hence , required probability = $P(A).P(B)$	
	$= \frac{625}{3888} \times \frac{1}{6} = \frac{625}{23328}$	1

61. An

Ans

$$\text{Here, } Z=\frac{X-42}{24}$$

(i) when $X=60$, we obtain $Z=\frac{60-42}{24}=\frac{3}{4}=0.75$ ½

$\therefore P(X>60)=P(Z>0.75)$

$=0.5-P(0\le Z\le 0.75)$ ½

$=0.5-0.2734$

$=0.2266$ ½

Thus, 226 (or 227) students scored more than 60 marks. ½

(ii) when $X=30$, we obtain $Z=\frac{30-42}{24}=-0.5$ ½

when $X=60, Z=0.75$

$\therefore P(30<X<60)=P(-0.5<Z<0.75)$

$=P(-0.5<Z<0)+P(0\le Z\le 0.75)$ 1

$=P(0\le Z\le 0.5)+P(0\le Z\le 0.75)$ ½

$=0.1915+0.2734$

$=0.4649$ ½

Thus, 464 (or 465) students scored marks between 30 and 60. ½

62. On

Q-32 Here m = 0.4

$P(X=r)=\frac{e^{-m}.m^r}{r!}=\frac{e^{-0.4}\times(0.4)^r}{r!}$

$=\frac{0.6703\times(0.4)^r}{r!}$ 1 Mark

In 1000 pages error = 1000 $\times$ $\frac{0.6703\times(0.4)^r}{r!}$ ½ Mark

For zero error $P(X=0)=1000\ \times\frac{e^{-m}.m^0}{0!}=\frac{e^{-0.4}\times(0.4)^0}{0!}$

$=1000\times 0.6703=670.3$ 1½ Mark

For one error $P(X=1)=1000\ \times\frac{e^{-m}.m^1}{1!}=\frac{e^{-0.4}\times(0.4)^1}{1!}$

$=670.3\times\ 0.4=268.12$ 2 Mark

63. How

Here p = $\frac{1}{2}$ $and\ q=\frac{1}{2}$

P(X=r) = C(n, r)$p^r q^{n-r}$

1-P(r = 0) > $\frac{90}{100}$ 1 Mark

$1- C(n,0)\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^n > \frac{9}{10}$

$\Rightarrow \frac{n!}{0!(n-0)!}\left(\frac{1}{2}\right)^n < \frac{1}{10}$ 2 Mark

$\Rightarrow \left(\frac{1}{2}\right)^n < \frac{1}{10} \Rightarrow 2^n > 10 \Rightarrow n\ is\ 4\ or$ more times 2 Mark

64. It

Probability of defective bucket = 0.03 n = 100 m = np = 100 x 0.03 = 3 Let X = number of defective buckets in a sample of 100 P (X = r) = $\frac{m^r e^{-m}}{r!}, r = 0,1,2,3, ...$	1
(i) P (no defective bucket) = P(r = 0) = $\frac{3^0 e^{-3}}{0!} = 0.049$	2
(ii) P (at most one defective bucket) = P(r = 0, 1) = $\frac{3^0 e^{-3}}{0!} + \frac{3^1 e^{-3}}{1!}$ $= 0.049 + 0.147$ $= 0.196$	2

65. In

X = scores of students, $\mu = 45, \sigma = 5$ $\therefore Z = \frac{X-\mu}{\sigma} = \frac{X-45}{5}$	1
(i) When X = 45, $Z = 0$ P (X > 45) = P (Z > 0) = 0.5 ⇒ 50% students scored more than the mean score	2
(ii) When X = 30, $Z = -3$ and when X = 50, $Z = 1$ P (30< X< 50) = P (-3< Z < 1) = P (-3< Z ≤ 1) = P $(-3 < Z \leq 0) + P\ (0 \leq Z < 1)$ = P $(0 \leq Z < 3) + P\ (0 \leq Z < 1)$ = 0.4987 + 0.3413 = 0.84 ⇒ 84% students scored between 30 and 50 marks	2

66. A

Sol. n= 100, p = $\frac{6}{100}$, m= np

Here m = $100 \times \frac{6}{100} = 6$.

$$P(r) = e^{-m}\frac{m^r}{r!}$$

(i) $P(0) = e^{-m}\frac{m^0}{0!} = e^{-6} = 0{\cdot}0024$ **(1)**

(ii) $P(2) = e^{-m}\frac{m^2}{2!} = e^{-6} \times \frac{36}{2} = 0.0432$ **(1)**

(iii)(a) $P(0) + P(1) = e^{-6} + e^{-6}\frac{m^1}{1!} = e^{-6} + 6e^{-6} = 7e^{-6} = 0{\cdot}0168$ **(1+1)**

OR

(iii)(b) Mean = Variance = m = np = 6 **(1+1)**

67. According

Sol.

(i) $\frac{k}{6} + \frac{2k}{6} + \frac{3(1-k)}{6} + \frac{4k}{2} = 1$ ½

$\Rightarrow 3 + 12k = 6$

$\Rightarrow k = \frac{3}{12} = \frac{1}{4}$ ½

(ii) Required Probability = P(X = 3) + P(X = 4)

$= \frac{3}{8} + \frac{1}{2}$ ½

$= \frac{7}{8}$ ½

(iii) (a)

X	0	1	2	3	4
P(X)	0	$\frac{1}{24}$	$\frac{1}{12}$	$\frac{3}{8}$	$\frac{1}{2}$

1

$E(X) = 1 \times \frac{1}{24} + 2 \times \frac{1}{12} + 3 \times \frac{3}{8} + 4 \times \frac{1}{2} = \frac{10}{3}$ weeks or $3\frac{1}{3}$ weeks **1**

OR

(iii) (b)

X	9600	12000	20000	50000
$P(X)$	$\frac{1}{24}$	$\frac{1}{12}$	$\frac{3}{8}$	$\frac{1}{2}$

1

$E(X) = 9600 \times \frac{1}{24} + 12000 \times \frac{1}{12} + 20000 \times \frac{3}{8} + 50000 \times \frac{1}{2} = ₹\,33900$ **1**

68. The

Here, $\mu = 75, \sigma = 8, n = 500$

(i) For $X = 75, Z = \frac{X-\mu}{\sigma} = \frac{75-75}{8} = 0$ ½

$P(X < 75) = P(Z < 0) = 0.5$

50 % of students scored below 75 marks. ½

(ii) For $X = 82, Z = \frac{82-75}{8} = 0.875$

$P((X > 82) = P(Z > 0.875) = 1 - P(Z < 0.875) = 1 - 0.8092 = 0.1908$ ½

$\therefore$ Required number of students $= 0.1908 \times 500 = 95.4 \approx 95$ ½

(iii) (A) For $X = 67, Z = \frac{67-75}{8} = -1$ ½

For $X = 83, Z = \frac{83-75}{8} = 1$

$P(67 < X < 83) = P(-1 < Z < 1)$

$= P(Z < 1) - P(Z < -1)$ 1

$= 0.8413 - 0.1587 = 0.6826$ ½

∴ Required number of students $= 0.6826 \times 500 = 341.3 \approx 341$

OR

(B) Top 10% corresponds to the 90th percentile.

$\Rightarrow Z = \frac{X-\mu}{\sigma} = 1.28$

$\Rightarrow \frac{X-75}{8} = 1.28$ 1

$\Rightarrow X = 85.24 \approx 85$ 1

∴ The minimum score required to qualify for the scholarship is 85 marks.

69. Let

x_i	0	1	2	3	4	5
$P(X=x_i)$	0.2	k	2k	2k	0	0

(i) Since $\sum P = 1 \Rightarrow 0.2 + k + 2k + 2k = 1 \Rightarrow 0.2 + 5k = 1 \Rightarrow 5k = 0.8$

$\Rightarrow k = \frac{4}{25}$ 1½ Mark

(ii) $P(X=2) = 2k = \frac{8}{25}$ 1 Mark

(iii) $P(X \geq 2) = 4k = \frac{16}{25}$ 1½ Mark

OR

$P(X \leq 2) = 0.2 + 3k = \frac{17}{25}$

70. According

a) $\frac{k}{6} + \frac{2k}{6} + \frac{3(1-k)}{6} + \frac{4k}{2} = 1 \Rightarrow k = \frac{1}{4}$ — 1

b) P (getting admission on applying at least 2 weeks ahead of application deadline)

= P (X = 2,3,4)

= $\frac{1}{12} + \frac{3}{8} + \frac{1}{2} = \frac{23}{24}$

[alternate method: 1 – P (X = 1) = 1 - $\frac{1}{24} = \frac{23}{24}$] — 1

c) X = week applied ahead of application deadline

X	1	2	3	4
P(X)	$\frac{1}{24}$	$\frac{1}{12}$	$\frac{3}{8}$	$\frac{1}{2}$
XP(X)	$\frac{1}{24}$	$\frac{1}{6}$	$\frac{9}{8}$	2

$\therefore$ E(X) = $\frac{80}{24} = 3\frac{1}{3}$ weeks — 2

OR

X = Scholarship money awarded for the week applied in, before the deadline

Week applied in	1	2	3	4
X	9600	12000	20000	50000
P(X)	$\frac{1}{24}$	$\frac{1}{12}$	$\frac{3}{8}$	$\frac{1}{2}$
XP(X)	$\frac{9600}{24}$	$\frac{12000}{12}$	$\frac{60000}{8}$	$\frac{50000}{2}$

$\therefore$ E(X) = ₹ 33,900