DIFFERENTIATION

ONE-MARK QUESTIONS

1. (B) $\frac{dP}{dt} = (\alpha - \beta)P$
2. (B) 100
3. (D) Rs 14
4. (D) $-8e^{-2x}$

5. (D) increasing in (0.5, 1) and decreasing in (0,0.5)
6. (D) Assertion (A) is false, but Reason (R) is true
7. (D) $\frac{d(AC)}{dx} = \frac{1}{x}(MC - AC)$
8. (d) Assertion (A) is false and Reason (R) is true.
9. (D) $\frac{y^2}{x\,(1 - y\log x)}$
10. (B) a>1
11. (D) neither maximum nor minimum value
12. (C) $\frac{d(AC)}{dx} = \frac{1}{x}(AC - MC)$
13. (B) 16
14. (D) 50
15. (B) 16
16. (C) $\left(0,\ \frac{1}{e}\right)$
17. (A) $\frac{-1}{16a}$
18. (A) The marginal cost decreases from 0 to 1 and then increases onwards.
19. (C) –10
20. (C) Assertion is true but Reason is false
21. (C) Rs 50
22. (C) (–∞, 0) and (0, ∞)
23. (A) $\frac{-1}{2at^3}$

[SQP 21-22]

24. (C) 100 units

[SQP 21-22]

25. (A) 33

[SQP 21-22]

26. A

Sol. Here, $6y = x^3 + 2$

$\Rightarrow 6\frac{dy}{dt} = 3x^2\frac{dx}{dt}$ $\left(\frac{1}{2}\right)$

As $\frac{dy}{dt} = 8\frac{dx}{dt}$, we have $\left(\frac{1}{2}\right)$

$48\cancel{\frac{dx}{dt}} = 3x^2\cancel{\frac{dx}{dt}} \Rightarrow x = 4, -4$ $\left(\frac{1}{2}\right)$

when $x = 4$, $y = 11$; when $x = -4$, $y = \frac{-31}{3}$. $\left(\frac{1}{2}\right)$

$\therefore$ Points on the curve are $(4, 11)$, $\left(-4, \frac{-31}{3}\right)$

27. If

Ans

$$y = \left(x + \sqrt{x^2+1}\right)^p$$

$$\Rightarrow y_1 = p\left(x + \sqrt{x^2+1}\right)^{p-1}\left[1 + \frac{2x}{2\sqrt{x^2+1}}\right] \qquad 1$$

$$= \frac{p}{\sqrt{x^2+1}}\left(x + \sqrt{x^2+1}\right)^p$$

$$= \frac{py}{\sqrt{x^2+1}}$$

$$\Rightarrow \left(x^2+1\right)y_1^2 = p^2y^2 \qquad ½$$

Differentiating both sides, we get

$$2xy_1^2 + 2\left(x^2+1\right)y_1y_2 = 2p^2yy_1$$

$$\Rightarrow xy_1 + \left(x^2+1\right)y_2 = p^2y$$

$$\Rightarrow \left(x^2+1\right)y_2 + xy_1 - p^2y = 0 \qquad ½$$

28. Find

Sol. $y = x^3 - 6x^2 + 9x - 8$

$$\Rightarrow \frac{dy}{dx} = 3x^2 - 12x + 9$$ **(1)**

$$\Rightarrow \frac{dy}{dx} = 3(x-1)(x-3)$$

Critical points are 1, 3 **(1)**

Showing, x=1 is a point of local maxima. $(\frac{1}{2})$

Showing, x=3 is a point of local minima. $(\frac{1}{2})$

29. Prove

Sol. $f'(x) = 2x - 1$ ½

$f'(x) = 0 \Rightarrow x = \frac{1}{2}$ ½

$\Rightarrow f'(x) > 0$ when $x > \frac{1}{2}$ and $f(x) < 0$ when $x < \frac{1}{2}$ ½ + ½

Since $f'(x)$ changes its sign in the interval $(-1,1)$, hence f(x) is neither strictly increasing nor strictly decreasing on $(-1, 1)$. **1**

30. Find

Sol. Let $P = y^x$, $Q = x^y$, $R = x^x$

Then, $\log P = x \log y \quad \Rightarrow \frac{dP}{dx} = y^x\left[\log y + \frac{x}{y}\frac{dy}{dx}\right]$ ½

$\log Q = y \log x \quad \Rightarrow \frac{dQ}{dx} = x^y\left[\frac{y}{x} + \log x \frac{dy}{dx}\right]$ ½

$\log R = x \log x \quad \Rightarrow \frac{dR}{dx} = x^x\,[1 + \log x]$ ½

$$\frac{dP}{dx} + \frac{dQ}{dx} + \frac{dR}{dx} = 0$$ ½

$$\Rightarrow \frac{dy}{dx} = -\frac{y^x \log y + yx^{y-1} + x^x\,(1 + \log x)}{x[y^{x-1} + x^{y-1}\log x]}$$ **1**

31. A

Given $y = px^3 + qx^2 \Rightarrow \frac{dy}{dx} = 3px^2 + 2qx$ (i)	
Since $x = 1$ is a critical point $\therefore 3p(1)^2 + 2q(1) = 0 \Rightarrow 3p + 2q = 0$ (ii)	1
Also, curve passes through (1, −1) so $-1 = p + q$ (iii)	
Solving (ii) and (iii) we get, $p = 2$ and $q = -3$	½
Using this in (i) we get $\frac{dy}{dx} = 6x^2 - 6x$	1
Now for other critical point, $\frac{dy}{dx} = 0 \Rightarrow 6x^2 - 6x = 0 \Rightarrow 6x(x-1) = 0$	
$\Rightarrow x = 0 \; or \; 1$	
Thus, the other critical point is (0,0)	½

32. The

$AR(x) = \frac{R(x)}{x} = 15 + \frac{x}{3} - \frac{x^3}{36}$, Now $\frac{d}{dx}AR(x) = 0 \Rightarrow 0 + \frac{1}{3} - \frac{3x^2}{36} = 0$	½
$\Rightarrow x^2 = 4 \Rightarrow x = 2$ (as x is non negative)	½
$\Rightarrow x = 2$ is the critical point of $AR(x)$.	½
Now $\frac{d^2[AR(x)]}{dx^2} = 0 - \frac{6(2)}{36} < 0$ hence $AR(x)$ is maximum at $x = 2$	½
$AR(2) = 15 + \frac{2}{3} - \frac{8}{36} = 15 + \frac{4}{9}$	½
Now, $MR(x) = 15 + \frac{2x}{3} - \frac{4x^3}{36} \Rightarrow MR(2) = 15 + \frac{4}{3} - \frac{8}{9} = 15 + \frac{4}{9}$	½
$\therefore AR(2) = MR(2)$	

33. Find

Q-28 f(x) = $2x^3 - 9x^2 + 12x - 5$	
$f'(x) = 6x^2 - 18x + 12 = 6(x^2 - 3x + 2)$	
$f'(x) = 6(x-1)(x-2)$	1 Mark
$f'(x) = 0 \Longrightarrow x = 1 \; and \; x = 2 \; are \; the \; critical \; points.$	½ Mark
The intervals are $(-\infty, 1)$; $(1,2)$; $(2, \infty)$	½ Mark
Increasing in $(-\infty, 1] \cup [2, \infty)$, Decreasing in $[1,2]$	1 Mark

34. Find

$f'(x) = x^3 - 6x^2 + 11x - 6 = (x-1)(x-2)(x-3)$ $\Rightarrow x = 1,2,3$	1
Strictly increasing in $(1,2) \cup (3,\infty)$	1
Strictly decreasing in $(-\infty,1) \cup (2,3)$	1

35. Find

[March, 2024]

Sol.

$f(x) = -\frac{3}{4}x^4 - 8x^3 - \frac{45}{2}x^2 + 105$

$f'(x) = -3x^3 - 24x^2 - 45x$ and $f''(x) = -9x^2 - 48x - 45$ **(1)**

$f'(x) = 0 \Rightarrow x = 0$ or $x^2 + 8x + 15 = 0$

i.e., $x = 0, -3, -5$ **(1)**

At $x = 0, f''(0) < 0 \Rightarrow 0$ is a point of local maxima **(1)**

At $x = -3, f''(-3) > 0 \Rightarrow -3$ is a point of local minima **(1)**

At $x = -5, f''(-5) < 0 \Rightarrow -5$ is a point of local maxima **(1)**

36. Find

Sol.

$f(x) = 20 - 9x + 6x^2 - x^3$

$f'(x) = -9 + 12x - 3x^2 = -3(x^2 - 4x + 3)$ **$(\frac{1}{2})$**

$= -3(x - 1)(x - 3)$ **(1)**

$f'(x) = 0 \Rightarrow x = 1,3$ **$(\frac{1}{2})$**

Now intervals are $(-\infty, 1)$, $(1,3)$ and $(3, \infty)$ **$(\frac{1}{2})$**

Intervals	Sign of $f'(x)$
$(-\infty, 1)$	$-$ve
$(1,3)$	$+$ve
$(3, \infty)$	$-$ve

$(1\frac{1}{2})$

$\Rightarrow f(x)$ is strictly increasing in $(1,3)$ or $[1,3]$ **$(\frac{1}{2})$**

And $f(x)$ is strictly decreasing in $(-\infty, 1) \cup (3, \infty)$ or $(-\infty, 1] \cup [3, \infty)$ **$(\frac{1}{2})$**

37. Divide

Sol. Let the two parts be x and 15 – x. Then, let $y = x^2(15 - x)^3$ **(1)**

$\Rightarrow \frac{dy}{dx} = x(15 - x)^2 (-5x + 30)$ **(1)**

$\frac{dy}{dx} = 0$ gives x = 0, 15, 6 **($1\frac{1}{2}$)**

Rejecting x = 0, 15. Hence x = 6

Showing, $x = 6$ is a point of maxima **(1)**

So, y is maximum when x = 6.

Hence two parts are 6 and 9 **($\frac{1}{2}$)**

38. Find

Sol. Let P (x, y) be the required point which is nearest to Q (1, 4). Then distance PQ should be minimum and hence $(PQ)^2$ should be minimum. **($\frac{1}{2}$)**

$$\text{Now, } (PQ)^2 = (x - 1)^2 + (y - 4)^2 = \left(\frac{y^2}{2} - 1\right)^2 + (y - 4)^2 \quad \textbf{(1)}$$

$$= \frac{y^4 - 32y + 68}{4} \quad \textbf{(1)}$$

Let $D = \frac{y^4 - 32y + 68}{4}$

$\frac{dD}{dy} = y^3 - 8$ **($\frac{1}{2}$)**

$\frac{dD}{dy} = 0 \Rightarrow y = 2$ **(1)**

Showing, y = 2 is a point of minima **($\frac{1}{2}$)**

Thus, the point is (2, 2) **($\frac{1}{2}$)**

39. A

Sol.	Let the length and breadth of rectangle be x and y units respectively.	
	Let L be the length of fencing required, then	
	$L = x + 2y$	**1**
	If 'a' denotes the area of the rectangular region. Then	
	$a = xy$	½
	$\Rightarrow L = x + \frac{2a}{x}$	½
	$\Rightarrow \frac{dL}{dx} = 1 - \frac{2a}{x^2}$	**1**
	$\frac{dL}{dx} = 0 \Rightarrow x = \sqrt{2a}$	**1**
	$\left(\frac{d^2L}{dx^2}\right)_{x=\sqrt{2a}} = \frac{2}{\sqrt{2a}} > 0$	½
	Thus L is least when $x = \sqrt{2a}$	
	Length of fence $= x = \sqrt{2a}$	

40. A

Ans	Let the length of the one piece bent into shape of a square be x m then the length of the other piece bent into the shape of a circle is $(36-x)$ m. Let side of square be 'a' and radius of circle be 'r'. Then,	
	$4a = x, 2\pi r = 36 - x \Rightarrow a = \frac{x}{4}, r = \frac{36-x}{2\pi}$	1
	Combined area $A = a^2 + \pi r^2$	
	$\Rightarrow A = \frac{x^2}{16} + \frac{1}{4\pi}(36-x)^2$	1
	$\frac{dA}{dx} = \frac{x}{8} - \frac{1}{2\pi}(36-x)$	1
	For maxima/minima, put $\frac{dA}{dx} = 0$	
	$\Rightarrow \frac{x}{8} = \frac{1}{2\pi}(36-x) \Rightarrow x = \frac{144}{\pi+4} m$	1
	Also, $\frac{d^2A}{dx^2} = \frac{1}{8} + \frac{1}{2\pi} > 0$ at $x = \frac{144}{\pi+4}$, so A is minimum.	½
	Lengths of two pieces are $\frac{144}{\pi+4} m$ and $\frac{36\pi}{\pi+4} m$ respctively.	½

41. Determine

Ans.

$f(x) = x^4 - \frac{x^3}{3} \Rightarrow f'(x) = 4x^3 - x^2 = x^2(4x-1)$

$f'(x) = 0 \Rightarrow \mathrm{x} = 0, \frac{1}{4}$ **1**

Thus, critical points divide R into three parts

$x < 0, 0 < x < \frac{1}{4}, x > \frac{1}{4}$ ½

When $x < 0, f'(x)$ is −ve

$\therefore f(x)$ is strictly decreasing for $x < 0$ **1**

When $0 < x < \frac{1}{4}, f'(x)$ is +ve

$\therefore f(x)$ is strictly increasing for $x < 0$ **1**

When $x > \frac{1}{4}, f'(x)$ is −ve

$\therefore f(x)$ is strictly decreasing for $x < 0$ **1**

Hence $f(x)$ is strictly increasing on $\left(\frac{1}{4}, \infty\right)$ and strictly decreasing on $(-\infty, 0) \cup \left(0, \frac{1}{4}\right)$ ½

42. A

Ans.

Here $C = \frac{x^3}{3} - 7x^2 + 111x + 50$ and $x = 100 - p$ i.e., $p = 100 - x$

(i) R, the revenue function is

$R = px = (100 - x)x = 100\,x - x^2$ ½

(ii) Profit function $P(x) = R(x) - C(x)$

$= (100\,x - x^2) - \left(\frac{x^3}{3} - 7x^2 + 111x + 50\right)$

$= -\frac{x^3}{3} + 6x^2 - 11x - 50$ **1**

(iii) $\frac{dP}{dx} = -x^2 + 12x - 11$ ½

For P to be maximum, $\frac{dP}{dx} = 0 \Rightarrow \mathrm{x} = 1, 11$ **1**

$\frac{d^2P}{dx^2} = -2x + 12 > 0$ at $x = 1$ and < 0 at $x = 11$

Thus P is maximum when $x = 11$ **1**

Hence, the profit maximising level of output is 11 units

(iv) Maximum profit $= [P(x)]_{x=11}$

$= -\frac{(11)^3}{3} + 6(11)^2 - 11(11) - 50$

$= 111.33$ *or* 111 **1**

43. A

Q-34. Let 'x' units of product be produced and sold. As selling price of one unit is Rs 8 total revenue on 'x' units = Rs 8x

(i) Cost Function C(x) = Fixed Cost + 25% of 8x

$= 24000 + \frac{25}{100} \times 8x$

= 24000 +2x. 1 ½ Mark

(ii) Revenue Function = 8x 1 Mark

(iii) Breakeven Point 8x = 24000+2x

x = 4000 1½ Mark

(iv) Profit function = R(x) – C(x) = 6x – 24000 1 Mark

44. The

Let x and y be the dimension of the printed pages then x.y = 180.
A = Area of the page = (x+4)(y+5)

$$= xy + 5x + 4y + 20$$

$$= 180 + 5x + 4\times\left(\frac{180}{x}\right) + 20$$

$$= 200 + 5x + \frac{720}{x}$$ 2½ Mark

For most economical dimension $\frac{dA}{dx} = 0 \Rightarrow 5 - \frac{720}{x^2} = 0.$

$$\Rightarrow x^2 = 144 \Rightarrow x = 12$$

Now $\frac{d^2A}{dx^2} = \frac{1440}{x^3}$

$\left(\frac{d^2A}{dx^2}\right)_{x=12} = \frac{1440}{12^3} > 0. \therefore A\ is\ minimum$

Hence, the most economical dimensions are 16cm and 20 cm 2½ Mark

45. An

	Let x be the number of guests for the booking Clearly, $x > 100$ to avail discount $\therefore$ Profit, P = $[4800 - \frac{200}{10}(x - 100)]\,x = 6800x - 20x^2$	2
	$\Rightarrow \frac{dP}{dx} = 6800 - 40x \Rightarrow x = 170$	1
	As $\frac{d^2P}{dx^2} = -40 < 0, \forall x$	1
	A booking for 170 guests will maximise the profit of the company And, Profit = ₹ 5,78,000	1

46. To

	P(x) = R(x) – C(x) = 5x – (100 + 0.025x²)	2
	⇒P′(x) = 5 – 0.05 x ⇒ $x = 100$	1
	As P′′(x) = −0.05 < 0, ∀ x	1
	∴ Manufacturing 100 dolls will maximise the profit of the company And, Profit = ₹ 1,50,000	1

47. A

Sol.

(i) $V = x\,(36 - 2x)^2$ **1**

(ii) $\frac{dV}{dx} = (36-2x)^2 + 2x(36-2x)(-2)$

$= (36-2x)(36-2x-4x)$

$= (36-2x)(36-6x)$

$= 12\,(18-x)(6-x)$ **1**

(iii) $\frac{dV}{dx} = 0 \implies x = 18$ or $x = 6$ **1**

Rejecting $x = 18$, we have $x = 6$

and $\frac{d^2V}{dx^2} = 12(18-x)(-1) + 12\,(-1)(6-x)$

$\implies \frac{d^2V}{dx^2} < 0$ at $x = 6$ ½

$\therefore$ volume is maximum for $x = 6$ ½

OR

(iii) $\frac{dV}{dx} = 0 \implies x = 18$ or $x = 6$ **1**

Rejecting $x = 18$, we have $x = 6$

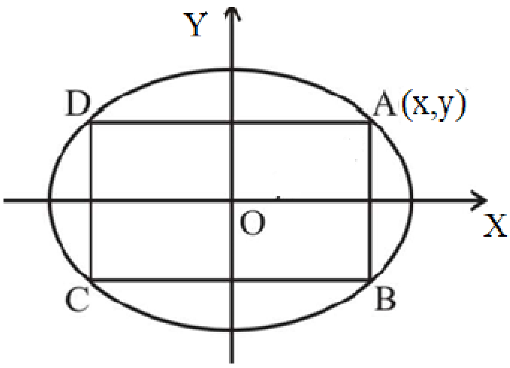
and $\frac{d^2V}{dx^2} = 12(18-x)(-1) + 12\,(-1)(6-x)$

$\implies \frac{d^2V}{dx^2} < 0$ at $x = 6$

$\therefore$ Max V $= 6\,(36-12)^2 = 6\,(24)^2 = 3456\ cm^3$ **1**

48. Read

Ans(i)



Area of soccer field $= (2x)(2y) = 4xy$ ½

$\Rightarrow A(x) = 4x \times \frac{b}{a}\sqrt{a^2 - x^2} = \frac{4bx}{a}\sqrt{a^2 - x^2}$ ½

Ans(ii)	$\frac{dA}{dx}=\frac{4b}{a}\left(\sqrt{a^2-x^2}+\frac{x(-2x)}{2\sqrt{a^2-x^2}}\right)$ or $\frac{dA}{dx}=\frac{4b}{a}\left(\frac{a^2-2x^2}{\sqrt{a^2-x^2}}\right)$ $\frac{dA}{dx}=0\Rightarrow x=\pm\frac{a}{\sqrt{2}}$ $\therefore$ Critical point is $x=\frac{a}{\sqrt{2}}$	½ ½
Ans (iii)(a)	$\frac{dA}{dx}>0$ for values of x close to $\frac{a}{\sqrt{2}}$ and to the left of $\frac{a}{\sqrt{2}}$ and $\frac{dA}{dx}<0$ for values of x close to $\frac{a}{\sqrt{2}}$ and to the right of $\frac{a}{\sqrt{2}}$ $\therefore A$ is maximum at $x=\frac{a}{\sqrt{2}}$. Length $=2x=\sqrt{2}\,a$ and breadth $=2y=\sqrt{2}\,b$	1 1
	OR	
Ans (iii)(b)	$\frac{d^2A}{dx^2}=\frac{4b}{a}\left[\frac{\left(\sqrt{a^2-x^2}\right)(-4x)-\left(a^2-2x^2\right)\left(\frac{-2x}{2\sqrt{a^2-x^2}}\right)}{\left(a^2-x^2\right)}\right]<0$ at $x=\frac{a}{\sqrt{2}}$ $\therefore A$ is maximum at $x=\frac{a}{\sqrt{2}}$. Length $=2x=\sqrt{2}\,a$ and breadth $=2y=\sqrt{2}\,b$	1 1

49. A

Ans.	(a) $y=40{,}00{,}000-(x-2000)^2$ Gives $\frac{dy}{dx}=-2(x-2000)$ So, $\frac{dy}{dx}=0$ when $x=2000$ $\frac{d^2y}{dx^2}=-2<0$ Hence y is max at $x=2000$ (b) Max Revenue $=40{,}00{,}000-(2000-2000)^2=40{,}00{,}000$ (c) Total Revenue $=40{,}00{,}000-(2500-2000)^2$ $=37{,}50{,}000$	**1½** **½** **1** **1**

50. A

(i) For all values of $x, y = x^2 + 7$

$\therefore$ Shivam's position at any point of x will be $\left(x, x^2+7\right)$

The measure of the distance between Shivam and Manita, i.e., D

$$D = \sqrt{(x-3)^2 + \left(x^2+7-7\right)^2} = \sqrt{(x-3)^2 + x^4}$$ ½ + ½

(ii) We have,

$$D = \sqrt{(x-3)^2 + x^4}$$

Let $\Delta = D^2 = (x-3)^2 + x^4$

Now,

$$\frac{d}{dx}(\Delta) = 2(x-3) + 4x^3 = 4x^3 + 2x - 6$$ ½

$$\frac{d}{dx}(\Delta) = 0 \Rightarrow x = 1$$ ½

(iii) (a): $\Delta''(x) = 8x^2 + 2$

Clearly, $\Delta''(x) = 8x^2 + 2 > 0$ at $x = 1$ 1

$\therefore$ Value of x for which D will be minimum is 1.

For $x = 1, y = 8.$

Therefore, required distance = $D = \sqrt{(1-3)^2 + (1)^4} = \sqrt{4+1} = \sqrt{5}$ 1

(iii) (b): $\Delta''(x) = 8x^2 + 2$

Clearly, $\Delta''(x) = 8x^2 + 2 > 0$ at $x = 1$ 1

$\therefore$ Value of x for which D will be minimum is 1.

For $x = 1, y = 8.$ 1

Thus, the required position for Shivam is $(1, 8)$ when he is closest to Manita.