

Modulo, Allegations & Mixtures, Speed distance & time, Races & Games

1. (B) 4
2. (A) 3 : 2
3. (B) 20 m
4. (C) 491 seconds
5. The correct answer is 3 am. After modification of options. In the board paper correct answer was not given and grace mark was provided to all students.

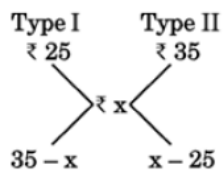
Sol.	Since correct answer is not in the options given So, it is suggested that 1 mark may be given to all who attempted this question	(1)
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6. (C) 9 am
7. (C) 6
8. (C) 6
9. (b) 8 : 7
10. (c) 8
11. (a) 2 km/h
12. (C) 2
13. (c) 31
14. (b) 2 km/h
15. (c) Truck A, 10 litres
16. (c) 3:5
17. (C) 7
18. (C) 5 km/h
19. (D) 35
20. (C) 25 meters
21. (B) 9:10 am
22. (A) 200 kg
23. (D) 120 hrs
24. (B) 7
25. (A) 2 km/h
26. (A) 1 : 6
27. (B) 28 m
28. (D) 2
29. (C) 190 seconds
30. 1.5 km/h
31. Solution:

Sol.	Let pipe Q fills the tank in x minutes, then P will fill the tank in $x - 15$ minutes.
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$\frac{1}{x} + \frac{1}{x-15} = \frac{1}{10}$	1
$\Rightarrow x^2 - 35x + 150 = 0$	$\frac{1}{2}$
$\Rightarrow x = 30$	$\frac{1}{2}$
$(x = 5 \text{ rejected})$	
$\therefore Q \text{ can fill the tank alone in 30 minutes.}$	

32. Solution:

Sol.	Let the price per kg of mixture be ₹ x	
		
	$\therefore \frac{35-x}{x-25} = \frac{3}{2}$	1
	$\Rightarrow 70 - 2x = 3x - 75$	
	Solving, we get $x = 29$	1
	$\therefore \text{Price per kg of the mixture is ₹ 29}$	

33. Solution:

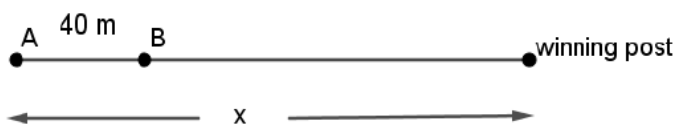
Ans	$(137 + 995)(\text{mod } 12)$ $\equiv 137(\text{mod } 12) + 995(\text{mod } 12)$ $\equiv 5(\text{mod } 12) + 11(\text{mod } 12)$ $\equiv (5 + 11)(\text{mod } 12)$ $\equiv 16(\text{mod } 12)$ $\equiv 4(\text{mod } 12)$ Hence $(137 + 995)(\text{mod } 12) = 4$	1
		1

34. Solution:

Ans	Let us evaluate $12^{12} \pmod{10}$ We know that $12 \equiv 2 \pmod{10}$ $\Rightarrow 12^4 \equiv 2^4 \pmod{10}$ $\equiv 16 \pmod{10}$ $\equiv 6 \pmod{10}$ $\Rightarrow 12^{12} = (12^4)^3 \equiv 6^3 \pmod{10}$ $\equiv 216 \pmod{10}$ $\equiv 6 \pmod{10}$ Hence, the unit's digit of 12^{12} is 6.	1 1
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35. Solution:

Sol.



Let the winning post be x metres away from the starting point.

$$\therefore \frac{x}{3/2} = \frac{x-40}{1}$$

$$\Rightarrow \frac{x}{2} = \frac{3}{2} \times 40 = 60 \Rightarrow x = 120 \text{ metres}$$

1

1

36. Solution:

Sol.

Suppose A takes ' t ' seconds to run 1 km race. Then, B takes $(t + 30)$ seconds and C takes $(t + 30 + 15)$ seconds, i.e. $(t + 45)$ seconds.

We find A beats C by $(30 + 15)$ seconds = 45 seconds and it is given that A beats C by 180 metres.

$\therefore$ C runs 180 m in 45 seconds

$\left(\frac{1}{2}\right)$

$\Rightarrow$ C runs 1000 m in $\left(\frac{45}{180} \times 1000\right)$ seconds = 250 seconds.

(1)

$$\therefore t + 45 = 250 \Rightarrow t = 205$$

$\left(\frac{1}{2}\right)$

Hence, A takes 205 seconds to run 1 km

37. Solution:

Sol. Let B be closed after n minutes. Then, pipe A runs for 18 minutes and B runs for n minutes to fill the tank.

$$\therefore \frac{18}{24} + \frac{n}{32} = 1 \quad (1)$$

$$\Rightarrow \frac{3}{4} + \frac{n}{32} = 1 \Rightarrow n = 8. \quad (1)$$

Hence, pipe B must be closed after 8 min

38. Solution

Last digit can be obtained by division with 10.

required answer = $(2^{100} + 100!) \bmod 10$

$$2^5 \bmod 10 = 32 \bmod 10 \equiv 2 \bmod 10 \Rightarrow 2^5 \equiv 2 \bmod 10 \Rightarrow (2^5)^5 \equiv 2^5 \bmod 10 \equiv 2 \bmod 10$$

$$\Rightarrow (2^{25})^4 \equiv 2^4 \bmod 10 \equiv 16 \bmod 10 \equiv 6 \bmod 10 = 6$$

Also $100! = 100 \times 99!$ which is divisible by 10

$$\text{So, } 100! \bmod 10 = 0 \Rightarrow (2^{100} + 100!) \bmod 10 \equiv (6 + 0) \bmod 10 = 6$$

39. Solution

Time taken to drain full tank = x hours i.e., the time rate of drain the tank = $\frac{1}{x}$ units per hour

Time taken to fill the full tank is 2 hours i.e., the time rate of filling the tank = $\frac{1}{2}$ units per hour

Again, with the leakage, the pipe takes $2\frac{1}{3} = \frac{7}{3}$ hours to fill the full tank.

The rate of filling the tank along with the leakage will be = $\frac{3}{7}$ units per hour.

Now, according to question,

$$\left(\frac{1}{2}\right) - \left(\frac{1}{x}\right) = \left(\frac{3}{7}\right)$$

Solving, we get $x = 14$

Hence, 14 hours are required to drain the full tank.

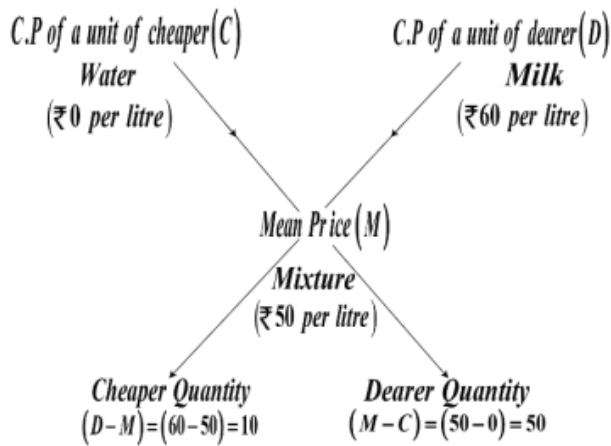
40. Solution

In a 200 m race, when A covers 200 m	
then B covers $(200 - 18) = 182\text{ m}$	
and C covers $(200 - 31) = 169\text{ m}$	
$\Rightarrow A : C = 200 : 169$	$\frac{1}{2}$
$\frac{B}{C} = \frac{A}{C} \times \frac{B}{A} = \frac{200}{169} \times \frac{182}{200} = \frac{182}{169}$	$\frac{1}{2}$
When B covers 182 m then C covers 169 m	
When B covers 350 m then C covers $\frac{169}{182} \times 350 = 325\text{ m}$	$\frac{1}{2}$
Therefore, B can give a start of $(350 - 325) = 25\text{ m}$ to C.	$\frac{1}{2}$

41. Solution

Sol.	Let the speed of the boat be $x\text{ km/h}$	
	$\therefore$ Speed of the boat upstream = $(x - 1)\text{ km/h}$	}
	and speed of the boat downstream = $(x + 1)\text{ km/h}$	
	$\therefore \frac{3.5}{x-1} + \frac{3.5}{x+1} = 1 + \frac{12}{60} = \frac{6}{5}$	$\frac{1}{2}$
	$\Rightarrow 3.5(2x)5 = 6(x^2 - 1)$	
	$\Rightarrow 6x^2 - 35x - 6 = 0$	$\frac{1}{2}$
	Solving, we get $x = 6\text{ km/h}$	$\frac{1}{2}$
	(rejecting the -ve value)	

42. Solution



C.P of 1 litre of water = ₹ 0

C.P of 1 litre of milk = ₹ 60

Mean Price = ₹ 50

Therefore, ratio of water and milk = $\frac{60-50}{50-0} = \frac{10}{50} = \frac{1}{5}$ or 1: 5

} 1

1

43. Solution

Speed of boat in still waters = x km/h

Speed of stream = y km/h

Distance travelled = d km

Time taken to travel downstream = $\frac{d}{x+y}$

Time taken to travel upstream = $\frac{d}{x-y}$

Then, $\frac{2d}{x+y} = \frac{d}{x-y} \Rightarrow x : y = 3 : 1$

1

1

44. Solution

Param runs 5 m in 3 seconds

$\Rightarrow$ time taken to run 200 m = $\frac{3}{5} \times 200 = 120$ seconds

Anuj 's time = $120 - 3 = 117$ seconds

1

45. Solution

Let speed of boat and stream be x km/h and y km/h respectively

$$(x + y) \times 5 = 25$$

$\frac{1}{2}$ Mark

$$(x - y) \times 5 = 15$$

$\frac{1}{2}$ Mark

$$\Rightarrow x + y = 5 \text{ and } x - y = 3$$

Solving, $x = 4$ and $y = 1$

$\frac{1}{2}$ Mark

So, speed of stream is 1km/h

$\frac{1}{2}$ Mark

46. Solution

When B runs 50 m A runs 40 m

$\frac{1}{2}$ Mark

When B runs 1 m , A runs $= \frac{40}{50} = \frac{4}{5}$

$\frac{1}{2}$ Mark

When B runs 1000 m , A runs $= \frac{4}{5} \times 1000 = 800$ m

$\frac{1}{2}$ Mark

Hence B beats A by 200 m

$\frac{1}{2}$ Mark

47. Solution

Let the total distance be d km and the speed of boat in still water be x km/h

Speed of stream = 5 km/h

Speed upstream = $(x - 5)$ km/h

$\frac{1}{2}$

Speed downstream = $(x + 5)$ km/h

$\frac{1}{2}$

According to question, $\frac{d}{x-5} = 3 \times \frac{d}{x+5}$

$\frac{1}{2}$

Solving, we get $x = 10$

$\frac{1}{2}$

Hence, the speed of boat in still water is 10 km/h

48. Solution

Sol.	Let the required time taken be n minutes	
	$\therefore \frac{1}{n} = \frac{1}{60} + \frac{1}{90}$	1
	$\Rightarrow \frac{1}{n} = \frac{3+2}{180} = \frac{5}{180} = \frac{1}{36}$	
	$\therefore n = 36 \text{ minutes or } \frac{3}{5} \text{ hours}$	1

49. Solution

53. Solution

Sol.	$7^2 = 49 \equiv -1(mod\ 10)$	
	$7^{295} = (7^2)^{147} \times 7$	1
	Now, $(7^2)^{147} \equiv (-1)^{147}(mod\ 10) \equiv -1(mod\ 10)$	1
	$\Rightarrow 7^{295} = (7^2)^{147} \times 7 \equiv -7(mod\ 10) \equiv 3(mod\ 10)$	$\frac{1}{2}$
	$\therefore$ Units digit is 3.	$\frac{1}{2}$

54. Solution

Sol.	Juice contained in the container after final replacement	
	$= 50 \left(1 - \frac{5}{50}\right)^5 = 50 \left(\frac{9}{10}\right)^5$	(2)
	$= 50 \times 0.59049 = 29.5$ litres	(1)

55. Solution:

Sol.	Let the original quantity of dettol be x litres and the quantity of Dettol replaced by water be y litres.	
	So, $y = \frac{x}{3}$. After 3 operations the quantity of dettol left $= x \left(1 - \frac{y}{x}\right)^3$.	(1)
	After 3 operations the quantity of water in the bottle $= x - x \left(1 - \frac{x}{3x}\right)^3$	(1)
	Hence, the required ratio is $x \left(1 - \frac{x}{3x}\right)^3 : [x - x \left(1 - \frac{x}{3x}\right)^3]$	
	$= \left(1 - \frac{1}{3}\right)^3 : \left[1 - \left(1 - \frac{1}{3}\right)^3\right]$	
	$= \frac{8}{27} : \frac{19}{27}$	
	$= 8 : 19$	(1)

56. Solution

Sol.

Here, $n_A = 3$, $n_B = 7$ and $n_C = 10$.

$$\frac{1}{n} = \frac{1}{n_A} - \frac{1}{n_B} - \frac{1}{n_C}$$

$$\Rightarrow \frac{1}{n} = \frac{1}{3} - \frac{1}{7} - \frac{1}{10}$$

$$\Rightarrow \frac{1}{n} = \frac{19}{210} \Rightarrow n = 11 \frac{1}{19}$$

Hence, the tank is filled in $11 \frac{1}{19}$ hours.

(2)

(1)

57. Solution:

Let speed upstream be x km/hr and speed downstream be y km/hr

Since distance upstream and downstream is same

$$\therefore 8\frac{4}{5}x = 4y \Rightarrow \frac{44}{5}x = 4y \Rightarrow \frac{y}{x} = \frac{11}{5} \quad (i)$$

$$\text{Now, speed of boat : speed of stream} = \frac{x+y}{2} : \frac{y-x}{2}$$

$$(i) \Rightarrow \frac{y+x}{y-x} = \frac{11+5}{11-5} \Rightarrow \frac{\frac{y+x}{2}}{\frac{y-x}{2}} = \frac{8}{3}$$

$$\therefore \text{speed of boat : speed of stream} = 8:3$$

1

1

1

58. Solution:

Let t minutes be the total time taken to fill the tank

So according to the question,

pipe A is open for $(t - 10)$ minutes, pipe C is open for $(t - 10)$ minutes, pipe B is open for 10 minutes.

Using work done per minute, we get

$$\frac{t-10}{30} + \frac{10}{60} + \frac{t-10}{120} = 1 \Rightarrow \frac{5t-30}{120} = 1$$

$$\Rightarrow 5t = 150 \Rightarrow t = 30 \text{ minutes.}$$

$\therefore$ It will take 30 minutes to fill the tank.

1

$1 + \frac{1}{2}$
 $\frac{1}{2}$

59. Solution:

$$5 \equiv 5(\text{mod } 7)$$

$$\Rightarrow 5^2 \equiv 25(\text{mod } 7)$$

$$\Rightarrow 5^2 \equiv 4(\text{mod } 7)$$

$$\Rightarrow 5^4 \equiv 4^2(\text{mod } 7)$$

$$\Rightarrow 5^4 \equiv 2(\text{mod } 7)$$

$$\Rightarrow 5^{20} \equiv 32(\text{mod } 7)$$

$$\Rightarrow 5^{20} \equiv 4(\text{mod } 7)$$

$$\Rightarrow 5^{60} \equiv 1(\text{mod } 7)$$

$$\Rightarrow 5^{61} \equiv 5(\text{mod } 7)$$

Hence, the remainder when 5^{61} is divided by 7 is 5

1

1

1

60. Solution:

Sol. Let the cistern be emptied in n hours after 5 a.m.

Clearly pipes A and B fill the cistern for n and $n - 1$ hours respectively, while pipe C empties the tank for $n - 2$ hours

$$\therefore \frac{n}{3} + \frac{n-1}{4} - \frac{n-2}{1} = 0$$

Solving, we get $n = \frac{21}{5}$

i.e., 4 hours 12 minutes past 5 am i.e., at 9:12 am

$(1\frac{1}{2})$

$(1\frac{1}{2})$

(1)

(1)

61. Solution:

(i)

A + B fill the tank in 6 hrs

B + C fill the tank in 10 hrs

A + C fill the tank in $\frac{15}{2}$ hrs

$$2(A + B + C) = \frac{6 \times 10 \times \frac{15}{2}}{6 \times 10 + 6 \times \frac{15}{2} + 10 \times \frac{15}{2}} = \frac{450}{60 + 45 + 75} = \frac{450}{180} = \frac{5}{2} \text{ hrs}$$

Hence A, B and C together will fill the tank in 5 Hrs

2 Mark

(ii) A will in $[(A+B+C) - (B+C)] = \frac{10 \times 5}{10-5} = 10 \text{ hrs}$

1 Mark

(iii) B will fill in $\frac{\frac{15}{2} \times 5}{\frac{15}{2}-5} = 15 \text{ hrs}$

1 Mark

OR

C will fill in $\frac{5 \times 6}{6-5} = 30 \text{ hrs}$

62. Solution:

Ans(i)	Part of the tank filled by pipes A and B in one hour $= \frac{1}{10} + \frac{1}{12} = \frac{11}{60}$	$\frac{1}{2}$
	So, two pipes will fill the tank in $\frac{60}{11}$ hours.	$\frac{1}{2}$
Ans(ii)	Part of the tank filled by pipes A and C in one hour $= \frac{1}{10} - \frac{1}{15} = \frac{1}{30}$	$\frac{1}{2}$
	So, two pipes will fill the tank in 30 hours.	$\frac{1}{2}$
Ans (iii)(a)	Part of the tank filled by three pipes A, B and C in one hour $= \frac{1}{10} + \frac{1}{12} - \frac{1}{15} = \frac{7}{60}$	$1\frac{1}{2}$
	So, these three pipes will fill the tank in $\frac{60}{7}$ hours.	$\frac{1}{2}$
Ans (iii)(b)	Let the pipe B be turned off after 't' hours.	
	Part of tank filled by A in 6 hours $= \frac{6}{10}$	}
	Part of tank filled by B $= 1 - \frac{6}{10} = \frac{4}{10}$	
	$\Rightarrow \frac{t}{12} = \frac{4}{10}$	$\frac{1}{2}$
	$\Rightarrow t = 4.8$	$\frac{1}{2}$
	$\therefore$ Pipe B is turned off after 4.8 hours	

63. Solution

a)	Pipe C empties 1 tank in 20 h $\Rightarrow 2/5$ th tank in $\frac{2}{5} \times 20 = 8$ hours	1
b)	Part of tank filled in 1 hour = $\frac{1}{15} + \frac{1}{12} - \frac{1}{20} = \frac{1}{10}$ th $\Rightarrow$ time taken to fill tank completely = 10 hours	1
c)	At 5 am, Let the tank be completely filled in 't' hours $\Rightarrow$ pipe A is opened for 't' hours pipe B is opened for 't-3' hours And, pipe C is opened for 't-4' hours $\Rightarrow$ In one hour, part of tank filled by pipe A = $\frac{t}{15}$ th part of tank filled by pipe B = $\frac{t-3}{15}$ th and, part of tank emptied by pipe C = $\frac{t-4}{15}$ th Therefore $\frac{t}{15} + \frac{t-3}{12} - \frac{t-4}{20} = 1$ $\Rightarrow t = 10.5$ Total time to fill the tank = 10 hours 30 minutes OR 6 am, pipe C is opened to empty $\frac{1}{2}$ filled tank Time to empty = 10 hours Time for cleaning = 1 hour Part of tank filled by pipes A and B in 1 hour = $\frac{1}{15} + \frac{1}{12} = \frac{3}{20}$ th tank $\Rightarrow$ time taken to fill the tank completely = $\frac{20}{3}$ hours Total time taken in the process = $10 + 1 + \frac{20}{3} = 17$ hour 40 minutes	2